The Effect of Strategic Behavior on Group Decisions

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According to HARSANYI, the *social utility* of an alternative in a group decision voting game is the sum of all voter utilities for that alternative. With random member utilities, the *expected social utility* is defined as the mean of the random variable *social utility*. The *social utility* for a decision rule depends on the number of alternatives, group members, the distribution of member utilities, and strategic considerations of the group members.

In this paper, the effect of strategic voting behavior on the *expected social utility* is computed and compared for three well known voting procedures: The Single Vote-Rule, the Borda-Rule and the Hare-Rule. It is assumed that each member has complete information about the others' utilities as *common knowledge*, and that individual strategic considerations lead to a HARSANYI-SELTEN-equilibrium as the outcome strategy combination of the voting game. According to the HARSANYI-SELTEN-Theory, the resulting equilibrium depends also on the members' prior distributions about other members' voting behavior. The effect of different of such priors is also discussed.

Introduction

In a group decision problem, the group members have to choose collectively one out of a finite number of alternatives. Usually conflicting objectives and information between members result in different preferences for the alternatives. Hence, a voting procedure has to be introduced in order to make a choice. Following ARROW's well known result about the impossibility of constructing a voting mechanism satisfying certain plausible conditions, theoretical and practical characteristics and fallbacks of many voting procedures have been discussed in the social choice literature. (e.g. ARROW [1951], BLACK [1958], SEN [1970], SCHAUENBERG [1978], [1992], MERRILL[1984]).

According to another well known result of GIBBARD [1973] and SATTERTHWAITE [1975], for every nondictatorial decision rule there exist situations in which rational voters can influence their payoff beneficially by voting strategically (e.g. by hiding their true preferences).

Although general properties of several decision rules concerning strategic considerations of voters have been discussed (e.g. MILLER[1973], NURMI[1984], [1987]), most of the existing analysis seems to be qualitative. This may be due to the difficulties of determining a unique outcome of the strategic anticipations. From a game theoretical point of view, the problem is to predict the outcoming equilibrium of the voting game. In this paper, the voting procedure is looked at as an uncooperative game, and equilibrium selection theory according to HARSANYI/SELTEN [1988] is used to model the process of reciprocal strategic anticipations of the group members in order to determine the outcome of the group decision problem.

Apart from strategic considerations, the group decision is influenced by the number of alternatives and group members, the distribution of individual preferences and the decision rule. In order to *quantify* effects of changes in these variables on the group decision, a measure for the quality of the decision has to be introduced.

We define the *social utility* of a group decision according to HARSANYI [1977] as the sum of all voter utilities for the chosen alternative, keeping in mind the problem of interpersonal comparisons between individual utilities. Treating the member preferences as random, the *social utility* of a decision is a random variable depending on the member preferences, and the *expected social utility* is defined as the mean of this random variable. Since only ratios between individual utility differences matter, preferences are normed in the interval [0;1] for every member.

During the whole analysis, the preferences for all alternatives in between are assumed to be equally distributed on the interval [0;1]. (*The R[0,1]-distribution maximizes statistical entropy between all distributions on this interval, see e.g. CHAN [1971], p. 1752f, STRELEN [1986] p. 5f. Hereby, the minimum amount of prior knowledge about the preference distribution between the maximum and the minimum preference is entertained*).

Modeling Strategic Behavior

The group voting procedure can be looked at as a game with M players, each maximizing his own payoff function, his preference for the chosen alternative, by choosing a voting strategy. The formal structure of a voting strategy depends on the decision rule. For the Single Vote-Rule, the set of voting strategies for each member equals the set of the numbers 1 to N, where N is the number of alternatives. For Hare-Rule and Borda-Rule, the set of voting strategies is the set of all permutations of numbers from 1 to N, since for these rules voting means ranking the alternatives. We assume simultaneous voting, and the only possible strategies are within voting itself. Especially it is assumed that negotiations between group members in order to build coalitions before voting do not exist. Hence, voting can be modeled as an uncooperative game as discussed.

Since there usually exist a lot of Nash-equilibriums in such voting games, an equilibrium selection procedure is needed in order to uniquely determine the outcome strategy combination and the chosen alternative.

For this reason, equilibrium selection theory according to HARSANYI and SELTEN [1988] is used to choose between equilibriums. Assuming that within the group, member preferences are known as *common knowledge*, the starting point for equilibrium selection is the Bayesian approach to decision making: every group member has a prior idea about the probabilities for the other members to choose a certain voting strategy. Optimal voting in the Bayesian approach for every member means to choose the strategy which maximizes the expected preference based on the priors. This behavior is called the *naive Bayesian* approach.

Usually, the *naive Bayesian* approach will not result in a game theoretically rational voting strategy, since the resulting strategy combinations will in a lot of cases not be a Nash-equilibrium: individual *naive Bayesian* strategies are best answers referring to the mixed strategy combinations which are consistent with the underlying individual priors, but not referring *to each other*.

With *naive Bayesian* behavior, members use only the information about each other's preferences (*first order information)*. However, they do not use the knowledge that the other members are rational, anticipating individuals themselves (*second order information,* see HARSANYI/SELTEN [1988] pp. 139-141).

HARSANYI/SELTEN equilibrium selection theory selects the Nash-equilibrium, which in a certain way is the most consistent equilibrium with the *naive Bayesian* behavior. Technically, this is achieved with the *tracing procedure*, which gradually feeds *second order information* into the members' expectations, until a Nash-equilibrium is reached. In the context of *expected social utility*, it is sufficient to use the *linear tracing procedure*, since the resulting equilibrium is unique for nearly all voting games (see HARSANYI/SELTEN [1988], p. 144). The computer simulation approximates the *linear tracing procedure* by moving t from t=0 to t=1 in ε-steps, with ε=0.1.

Prior Distributions

Rational prior distributions depend only on the ratios between each members' utilities for the alternatives. Additionally, some priors are irrational for certain decision rules: The probability that a specific member votes for his least preferred alternative must be 0 for the Single Vote-Rule, and the least preferred alternative must be on the last position in any ranking with positive probability for the Hare-Rule. Similarly, for the Borda-Rule no rational member will, even under strategic considerations, put the least preferred alternative in rank 1 or the most preferred alternative in the last rank.

Besides, for the Single Vote-Rule each members' prior distribution about another members' chosen alternative should be monotone in this members' preferences. The most obvious distribution equals the normalized preferences, and every positive monotone function from [0;1] onto itself determines a possible prior distribution. For reasons of simplicity, the 5 examples to the right have been analyzed as prior distributions for the Single Vote-Rule. With obvious meaning the priors can be said to be stronger polarizing from left to right.

For Hare-Rule and Borda-Rule, if two strategies differ just in the ranking of two alternatives, the strategy with the more preferred alternative on the higher rank should have no lower prior probability than the corresponding one. For the Hare-Rule, one plausible prior is calculated by using conditional probabilities proportional to the preferences, since the most important strategic decision for the member is which alternative to put on rank 1, than rank 2 and so on. For the Borda-Rule proportionality to the sum of alternative preferences weighted with the potential Borda-points leads to a plausible prior.

Again for simplicity, in this paper with three priors each for the Hare-Rule and the Borda-Rule are investigated:

- The prior probability of a voting strategy equals the same probability for all rational strategies
- The prior probability of a voting strategy equals the discussed plausible priors
- The prior probabilty for the naive strategy (ranking the alternatives according to the preferences) is 1, all other strategies get a prior probability of 0.

(See LINDSTÄDT [1995], pp. 128-160, for a more complete discussion of modeling strategic behavior and priors.)

Methodology

In the remaining section of this paper, three propositions concerning different aspects of influencing factors for the *expected social utility* of a decision problem with the before mentioned characteristics are made. Due to the restricted space reason for this propositions is given in the form of diagrams, which show the results of computer simulations. For the examples the reason is quite obvious. The more general propositions seem plausible from these examples. (For more detailed information see LINDSTÄDT [1995] in a similar context.)

<u>Proposition 1:</u> *Expected social utility without strategic behavior* (*see Fig. 2*)

(a) For a fixed number of group members, the change in *expected social utility* for an increasing number of alternatives depends on the voting procedure. With an increasing number of alternatives, the *expected social utility*

- decreases for the Single Vote-Rule
- stays nearly constant for the Hare-Rule (with small shifts up and down)
- increases for the Borda-Rule.

The maximum *expected social utility* as an upper bound when choosing the alternative with maximum *social utility* every time apparently also increases. Hence, the *expected social utility* ignoring strategic behavior increases with the degree of information about individual preferences, the voting procedure contains.

(b) The *expected social utility* difference between the investigated voting procedures increases absolutely but decreases relatively with growing group size. Looking at the *relative* difference (*per member*), the importance of the voting procedure decreases with growing size of the group.

Proposition 2: *Effect of strategic behavior on the expected social utility* (s*ee. Fig. 3*)

(a) Strategic behavior of the group members has a positive effect on the *expected social utility*. The gain in *expected social utility* caused by consideration of strategic behavior increases c.p. with

- a decrease in information about the individual preferences contained in the voting procedure.
- a growth in the number of alternatives.

(b) Hence, the influence of the voting procedure on *expected social utility* decreases when strategic behavior is taken into account.

(*Ranges indicate differences due to assumption of different prior distributions for the tracing procedure.*)

(*To a*) This does not mean, however, that the *social utility* of every single decision is affected in a positive way. Counter examples can be constructed easily. But the proposition holds in average at least for the investigated voting procedures.

(*To b*) This result indicates that implications of the voting procedure for the average group decision are practically of smaller importance than theoretically and for single decisions.

The assumption of naive Bayesian behavior of group members can be looked at as an approximation for strategic voting behavior. The quality of this approximation can be measured in terms of *expected social utility* difference or as the share of all cases in which naive Bayesian behavior is already strategically (i.e., game theoretically) rational (a Nash-equilibrium).

Proposition 3: *Approximation of strategic behavior with the naive Bayesian approach* (*see Fig. 4, 5*)

For the Single Vote-Rule and the investigated priors, the approximation improves for the stronger polarizing priors. However, this result does not even for the investigated three priors hold for the Hare-Rule and the Borda-Rule (*not shown*), where differences in *expected social utility* between naive and strategic behavior are smaller anyway.

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