Handelshochschule Leipzig (HHL)

Decisions of the Board

Hagen Lindstädt

HHL-Arbeitspapier Nr. 41

Abstract

This paper presents a model for investigating group decisions of managers who are imperfectly informed and averse to taking risks. Of primary interest is financial success for risk neutral shareholders who have no other aim than maximizing expected profit and can therefore be treated homogeneously with respect to their objectives. The managers have a share in the financial success of the decision as incentive and furthermore pursue personal goals. Following the managers' individual judgement of available courses of action and a discussion within the management group (the board), a vote is taken. This vote reflects the managers' strategic behaviour as a result of personal preferences and their tendency to conform. Eight factors that influence decision-making will be identified and their impact on decision quality being discussed: personal interests, incentives, qualification, group size, strategic voting behaviour, risk aversion, tendency to conform, and type of voting procedure.

Table of Contents

1 Issue: Management decisions as group decisions

In contrast to companies that are run by their owners, business decisions in companies run by management groups are made by groups of individuals (the managers) with different qualifications and heterogeneous objectives that may differ from those of the owners. The members of the management group will assess the quality of a decision differently, depending on their individual assessment of the situation and their personal goals. From an economic point of view, it is particularly interesting to see how the shareholders assess the quality of the decision made by the management group. If, as will be assumed in the following, the shareholders are only interested in the economic position of the company, then it follows that each of them will only be interested in the long-term financial success of the decision that he has delegated to the managers.

Implications and logical attributes of voting mechanisms from the perspective of the voters are investigated in Social Choice Theory. According to Arrow's well-known impossibility theorem, a voting mechanism that satisfies the four Arrow Axioms and could so aggregate individual preferences in a fair way doesn't exist [1, 19]. Furthermore, the influence of individual group members via strategic voting behaviour is investigated in the literature. Strategic considerations of the individual group members reflect the behaviour they anticipate of other group members with the aim of influencing the result of the vote in their own best interests. According to Gibbard and Satterthwaite, it may for certain combinations of individual preferences always be to the advantage of rational individuals to behave strategically, no matter which voting procedure is employed, as long as it is not dictatorial [3, 18]. The variety and complexity of the individual cases and phenomena make an analysis difficult, so that most investigations are concentrated on the analysis of the fundamental characteristics of individual procedures [11, 14].

In contrast, the shareholders are faced with either making a decision themselves or delegating it to a group of managers. By delegating to the board the shareholders can hope to reach better decisions, i.e. improving the companies profitability, because of managements' superior knowledge or qualification. On the other hand, each manager pursues his own goals which may differ from those of the shareholders. The goals of the managers can, however, be at least partly compensated for by an incentive system that offers them a share in the profit made when the owners' goal is achieved. Compensation by means of an appropriate incentive system of course generates reward costs for the owners and reduces their total gain.

The following metadecision problem results for the shareholders: They have to find an optimum way of matching incentives, group composition and rules of group decision (e.g., the voting procedure) with their own goals while taking qualification, conflicting goals and reward costs into consideration. In principle and provided that the shareholders know what information the group members have, this problem can be solved by employing Laux's delegation value concept [7, 8]. That the shareholders are so well-informed is, however, not usually the case.

This problem calls to mind the multiagent approaches of the agency theory [5, 12, 13] that deal mainly with the derivation of optimum incentive systems. These approaches generally stem from the assumption that a random, functional dependence exists between a level of activity related to the disutility of the agent's work and a profit-related output. The output depends on the *joint* level of activity of the agents and on stochastic factors. These random influences cannot be observed by the principal, or at least not without cost, and can result in the type of misbehaviour known as *moral hazard*. A distinction is usually made between random influences that are only observable by the individual agents (*idiosyncratic shocks*), and random influences that are identical for all agents (*common shocks*). These models are based on the idea that the individual agent's activities influence the total output or profit, while the agents are indifferent of the solution itself.

In making management decisions, however, it would seem that the level of activity of the managers is less of a critical factor than their qualification and the personal interest they attribute to the alternatives. A manager's decision in favour of an alternative may be associated with a gain in prestige or well-founded personal interests such as the broadening of his own field of activity or sphere of influence.

Therefore this paper presents a model for management decisions that, unlike the agency theory, takes the managers' personal interests in the alternatives into account. After the group of managers has been assembled and the go-ahead from the shareholders has been received, an autonomous decision is made without any further intervention. The managers evaluate the alternatives firstly on the basis of their own information and then on the basis of shared information. In so doing they behave in a rational, utility-maximizing manner. Due to personal interests in the alternatives, which are assumed to be independent of each other in the model, a conflict of aims between shareholders and managers exists, resulting in *moral hazard* as a consequence of uncertainty as to whether a manager picks an alternative on the basis of high profit expectation or because of his personal interests. Unlike the disutility of work in the agency theory however, the managers associate personal interests *with the alternatives themselves* and not with the disutility of work that would be associated with extracting information. A level of activity as in the agency theory is thus not necessary. Instead, distribution assumptions about personal interests are required.

To compensate for the conflict of aims due to personal interests, the managers are given an across-the-board share in the actual profit that is realized at a later date from the chosen alternative. This incentive generates reward costs and, along with personal interests, has an influence on the utility functions of the group members. Each shareholder's capital is presumably well enough diversified to justify the assumption that he is risk neutral. Therefore, all shareholders can be assumed to have the same linear utility function with respect to the decision's profit.

2 Formalizing the objectives of shareholders and managers

The model is based on a decision problem with known alternatives A_n, n=1,…,N. The profit expectations from the alternatives are however unknown to the shareholders and the managers. A more detailed description of the model is given in [10].

2.1 The shareholders' objective

The shareholders don't know the profit expectations $g(A_n)$ relevant to making a decision, otherwise they wouldn't have a decision problem. For this reason *these profit expectations for their part* are taken to be random values between the alternatives. Presumably the shareholders have a subjective, common idea of the interval [a;b] in which the profit expectations from all alternatives lie. Other than that they have no information about the underlying probability distribution of the profit expectations.

They therefore assume a prior distribution with maximum statistical entropy within the set of all distributions on [a;b], so choosing the assumption that is as *random*, *uncertain* and *most non-committal* as possible. Of all the distributions over a real interval, the rectangular distribution is the one with maximum entropy. Its density takes on the value 1/(b-a) over the interval [a;b], and the value 0 outside this interval [6]. The shareholders thus assume that, in accordance with the principle of maximum entropy of prior distribution, the profit expectations of the given alternatives are stochastically independent and rectangularly distributed over the real interval [a;b], with a mean profit expectation of $E_{Gew}(g) = (a+b)/2$ and a profit expectation range of $d_{Gew}(g) = b-a$.

In the metadecision problem the shareholders maximize the profit expectation minus the cost of reward. At this point in time the profit that is actually realized from the alternative is not observable. The relevant utility to the shareholders in determining the control variables is the profit *expectation* $g(A_n)$ from the alternative n that the board chooses with these control variables. The expected utility of the random selection process (metadecision problem) is the mean profit *expectation* from the chosen alternative (original decision problem). An expected value is thus first formed from the uncertain profit from the individual alternatives and then second by the random sampling process.

The real number *Quality* (A_{n*}) describes the utility for the shareholders with the metadecision problem in deciding in favour of A_{n*} after deduction of reward costs and suitable rescaling. The quality of the decision in favour of A_{n*} is defined as an improvement in profit expectation $g(A_{n*})$ in deciding for n* compared with the smallest possible profit expectation $a = [E_{Gew}(g)-d_{Gew}(g)/2]$. The sum of the rewards for all M group members $B_m(A_{n*})$ will be subtracted from the difference in profit expectation. The result will then be compared with the maximum possible improvement, thus with the range of distribution $d_{Gew}(g)$. This positive linear transformation that normalizes the utility function does of course not influence the shareholders' decision.

(1) $$Quality(A_{n*}) \;:=\; \frac{g(A_{n*}) - \sum_{m=1}^{M} B_m(A_{n*}) - \left(E_{Gew}(g) - \frac{d_{Gew}(g)}{2}\right)}{d_{Gew}(g)}$$

An alternative definition relative to the *actual* range between maximum and minimum profit expectation in a single (simulated) decision is unsuitable because different ranges can even be obtained for different decisions belonging to the same distribution. If one management group should be better at making its decisions over small, *actual* ranges, while another group is better at making a decision in a more important case over a greater range, then the quality of a decision made by the first-mentioned group would be overestimated when using the alternative definition.

Individual alternatives are generalized in the shareholders' metadecision problem. The shareholders only know parameters of the metadecision problem in the form of a vector of control variables v and a vector of situational parameters p, which have random influence on the decision and therefore on its quality. The realization of a particular quality is taken as the reflection of a real valued stochastic variable that assigns to every causative reality dependent on p and v the decision in favour of an alternative A_{n*}, and thus the resultant quality.

Distribution and expected quality are of interest in the metadecision problem. In a problem characterized by (v;p), the distribution of the decision quality has an unknown distribution function H(Quality | (v;p)). As explained in the metadecision problem, the risk neutral shareholders behave rationally if they maximize the *expected* decision quality (EDQ) by choosing control variables v at given situational parameters p. As already mentioned, the analytical calculation of EDQ or even of distribution H can be quite difficult. However, EDQ can be approximately calculated using Monte Carlo simulations.

2.2 The managers' objectives

The managers pursue financial and personal interests. To differentiate between the bonuses of managers in a heterogeneous group cannot be to the shareholders' systematic advantage assuming they only know the distribution of managers' characteristics' within the group but nothing about their individual qualities. $B_m(A_n)$ represents the financial reward of manager m after choosing alternative n. A linear reward B of the kind $B = f \cdot G + F$ is assumed. G represents the *actual* profit associated with the realization of an alternative and $f \in [0;1]$ the bonus rate. The optimality of bonus rates determined and discussed later in this article is only guaranteed for this class of linear reward functions. As F is a fixed quantity and not a decision variable, and the managers are assumed to be either risk neutral or later to have a constant absolute risk aversion, it can be assumed that F=0. In the case of risk neutrality and the absence of personal interests, the resulting utility function U_m is well known to be linear with the expected reward $f \cdot g(A_n)$ as certainty equivalent.

It is further assumed that the utility associated with personal interests can be expressed as a definite, equivalent cash payment. The value of the utility associated with personal interests is assumed to be independent of the bonus f and is described as $PI_m(A_n)$ for group member m and alternative n, thus assuming a multiplicative model. With knowledge of the real interval for $PI_m(A_n)$ and in accordance with the principle of maximum entropy, the shareholders again assume stochastically independent rectangular distributions between the managers. The managers substitute estimated values $g_m(A_n)$ for the unknown, actual $g(A_n)$, as described in the next section. The preference function V_m for risk neutral manager m can be derived from his linear utility function.

(2) $$V_m(A_n) = f \cdot g_m(A_n) + PI_m(A_n) .$$

An analytical solution of the model is hardly feasible. On the one hand, the joint distribution of the group members' preferences and profit expectations from the alternatives irrespective of the bonus rate would have to be ascertained from assumptions about the distribution of personal interests and errors of estimation prior before and following the interaction. This would later supplement the influence of risk aversion and tendency to conform. The distribution of the profit expectations would then have to be calculated as the distribution that is induced by the random variable voting. In addition, the treatment of strategic behaviour in the voting process with its usually multiple Nash equilibria is very difficult.

In order to get round these difficulties, calculations are made with the help of a simulation model. Each of the parametric constellations described below is based on a Monte Carlo simulation with 5000 runs, or 3000 runs for investigations into strategic behaviour. The given values are approximations of actual, very hard to calculate values. However, the simulation procedure and sample sizes used indicate that the approximations are very reliable at least within the scope of the qualitative statements made. For statements about the EDQ involving incentives, the optimum bonus rate f_{opt} usually depends on the other variables and situational parameters, so that it has to be calculated for each data point in advance. This is done by running the simulation for multiple, sufficiently close bonus rates.

3 The board's decision-making process

The decision-making process consists of three steps. First of all, the managers individually form subjective beliefs about the profit distribution of the alternatives and opinions about their personal interests. In estimating profit distribution they make normally distributed, unbiased mistakes depending on their qualification. The managers then communicate these beliefs to each other in an interactive process, thereby improving the average quality of their estimations. Finally, they vote secretly and strategically on the basis of their complete, individual preferences for the alternatives. Strategic behaviour is modelled on Harsanyi and Selten's theory of equilibrium selection. A group that votes using the single-vote criterion is used as reference: each manager has one vote that he can cast for one of the N alternatives. The alternative with the most votes is selected.

3.1 Managers' individual estimation of the alternatives

In order to behave rationally in accordance with his preferences, each manager forms an opinion at the beginning of the decision-making process about the profit distribution to be expected from the alternatives. In this process, which will be referred to below as estimation, each manager m forms the subjective belief $g_m(A_n)$ about the profit expectations. In these estimations deviations from actual values occur, i.e. there is a prediction conflict in addition to the conflict of aims due to personal interests.

The quality of an estimation is reflected in predictive power and can be assessed from the deviation from an actual value. To make sense, individual deviations in the model have to be regarded as random. Justified by the central limit theorem a normal distribution is assumed to be the distribution of errors of estimation because, according to a frequently articulated idea, errors of estimation and measurement are caused by many additive, independent disturbance variables. Systematic misrepresentations can occur as a result of the interdependency of individual estimations. The shareholders are however unable to anticipate such interdependencies and misrepresentations operationally. For this reason stochastically independent, unbiased errors of estimation (expected value 0) are optimistically assumed for the managers in this model.

The managers have a predictive power that is parameterized as the standard deviation $s_{Estimate}$ of the error distribution. Accordingly, the standard deviation of estimation is small when the predictive power is good, and big when the predictive power is bad. Thus $g_m(A_n) \sim N(g(A_n); s^2_{Estimate})$, stochastically independent for all m, n. The assumption of on average correct, stochastically independent estimations of profit distribution by managers is optimistic and means that the bigger the management group the better its predictive power.

3.2 Board discussion

After the managers have formed individual opinions about the alternatives, a discussion of the board follows in which each manager gives, receives and interprets information, subject to his personal goals. It is assumed that, prior to this interaction, the utility functions and personal interests but not the estimated profit distributions of each manager are *common knowledge*. Each manager pursues two goals in the interaction: to influence the other group members in accordance with his own preferences and to improve the predictive power of the group. In addition the managers can try to change each others personal interests and estimations of profit expectation.

Due to its complexity, the interaction process has to be greatly simplified in the model. It is assumed that the group members always receive information simultaneously. The managers will often see through an *explicit* influencing of their personal interests as manipulation. Nevertheless, it is to be expected that personal interests will change during the interaction. *Systematic* changes in personal interests are however just as unpredictable from an operational point of view as their original distribution. For this reason the *distribution* of personal interests after the interaction remains unchanged, although this may not necessarily be true for single values $PI_m(A_n)$.

Information is not only exchanged about personal interests but also about estimated values $g_m(A_n)$. In order to make things less complex, it is assumed that the managers do not misrepresent their estimations intentionally because by doing so they would be putting their credibility at stake. While small misrepresentations are not of any real importance, bigger misrepresentations can damage a board member's credibility.

To withhold estimated values is then not a rational thing for a manager to do: Although the manager may be aware of his limited qualification, he has no rational way of knowing whether his estimation is higher or lower than the estimations of the other managers, and thus what kind of an influence it would have on them. Knowing his own qualification does not help him to improve his own estimation. On the other hand, by making his estimation known a board member increases the predictive power of the group and thus the chance of realizing the value he prefers on the basis of the subjective information available to him c.p. Hence, in the model each manager is truthful in the interaction about his $g_m(A_n)$.

How can managers use the estimations of other managers to their best advantage? In view of their levels of information, the best thing for them to do would be to try to minimize the *distribution* of errors of estimation in a certain way, as the errors of estimation themselves are unknown. In order to do this, the distributions must be evaluated using a loss function. It is usual and meaningful to look only at unbiased estimators and convex loss functions. If the qualification of all managers is identical like assumed here, it is optimal to determine the arithmetic mean of all the individual estimations, thus weighting all estimations equally. All managers then have identical estimations of $g(A_n)$ after the interaction [2], resulting in the following distribution for all n and m [9].

$$(3) \qquad \frac{\sum_{m=1}^{M} g_m(A_n)}{M} \sim N\left(g(A_n);\left(\frac{s_{Estimate}}{\sqrt{M}}\right)^2 \right)$$

3.3 Voting of the board

Finally, the managers vote according to the single-vote criterion. They behave rationally and strategically by anticipating the voting behaviour of the other board members. Theoretically, the voting can be regarded as a game with M players who each try to use their votes to maximize the their payoff, consisting of personal interests and of estimations for expected profit, adapted in the course of the interaction. For the single-vote criterion, the strategic space of a group member corresponds to the feasible set {1,...,N}. It is assumed that all group members vote secretly and at the same time, with no negotiations, consultations or side payments. The only moves allowed in the game are the votes. This means the vote can be modelled as an uncooperative nonzero sum game. Secret ballot is necessary for the managers to be able to make their estimations known in the interaction without having to fear the consequences for having voted in a particular way.

Normally, there are numerous Nash equilibria in such games so that a selection method is required [15, 16]. Selection is here made according to Harsanyi and Selten [4]. Their selection procedure starts with the basic idea that the preferences of all managers are *common knowledge* within the board and that, on the basis of this knowledge, each manager forms his own prior beliefs as to how the other managers are going to vote. In forming this prior opinion strategic considerations are not yet taken into account. In this approach, each manager votes for the alternative that maximizes his expected preference, which was constructed on the basis of his prior beliefs about the distribution of the other managers' voting behaviour. Harsanyi and Selten call this type of behaviour *naïve Bayesian*.

Naïve Bayesian behaviour does not generally result in a Nash equilibrium, as would be required by the usual rationale of game theory: every naïve Bayesian strategy is the best answer to that combination of mixed strategies that constitutes the prior opinion formed by a board member. The strategies are, however, not generally the best answers *to each other*. That all players follow strategies that are the best answers *to each other* is, however, characteristic of Nash equilibria. In naïve Bayesian behaviour, the members only use information about the preferences of the other group members but not knowledge about their capability to rationally anticipate.

The Harsanyi/Selten model selects the equilibrium that is in a certain sense most consistent with or the least different to naïve Bayesian behaviour. This is achieved technically by means of the so-called tracing procedure, which integrates the information about the rationality of the other players bit by bit into the expectations of the individual players until a Nash equilibrium is reached. Using the “linear tracing procedure” is sufficient for calculation of the expected value EDQ because the equilibrium is almost always unequivocal in the stochastical sense [4]. In the Monte Carlo simulation, the linear tracing procedure is approximated by increasing a parameter t in stages of 0.1.

In big groups naïve Bayesian behaviour may however often result directly in a Nash equilibrium. Indeed, a vote in which no *single* group member is able to change the result by changing his vote is already a Nash equilibrium. Strategic considerations over and above the Bayesian calculus are therefore only of importance in narrow majorities. The EDQ for naïve Bayesian behaviour is quite often a good approximation of that for strategic behaviour.

The choice of a voting strategy by individual managers depends on their prior estimations of the distribution of the other managers' voting behaviour. These may be rational or irrational, whereby even the rational estimations may be of varying plausibility. Rational prior estimations of distribution only depend on relative preferences and not on absolute values. The investigation can thus be made with normalized preferences $V^*_m(A_n)$ over the interval [0;1], where for all m and n we take

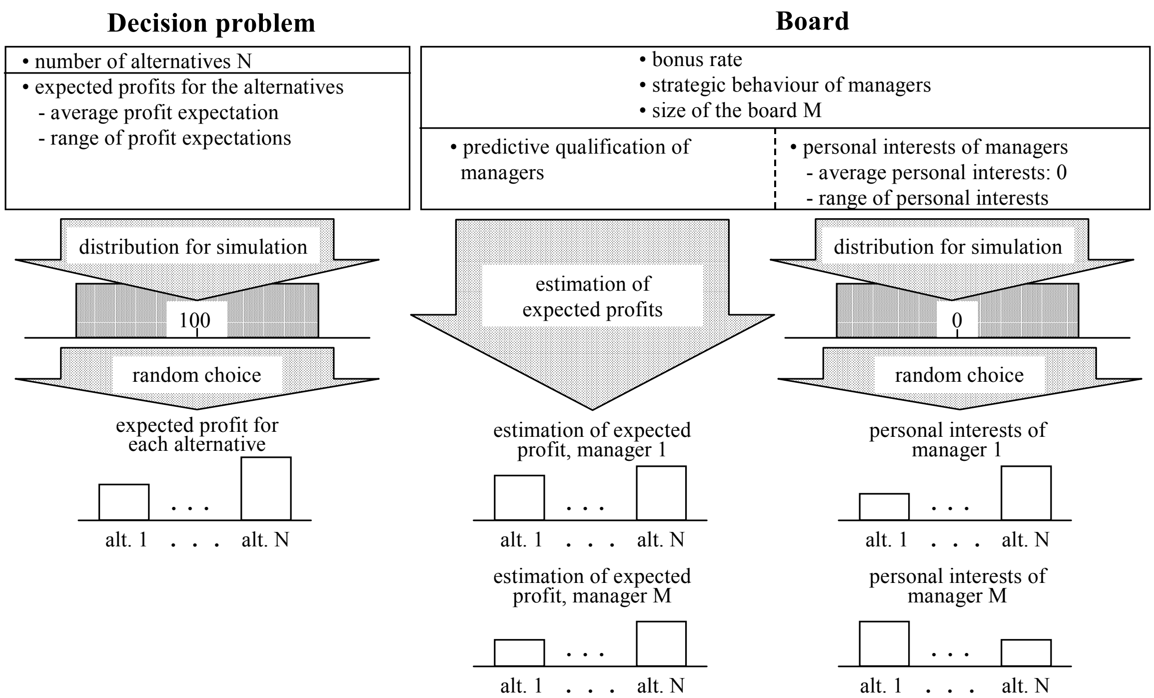
$$(4) \qquad V_m^*(A_n) \;:=\; \frac{V_m(A_n) - \min\limits_{l=1,\dots,N}\{V_m(A_l)\}}{\sum\limits_{k=1}^{N}\left(V_m(A_k) - \min\limits_{l=1,\dots,N}\{V_m(A_l)\}\right)}$$

When voting according to the single-vote criterion, no rational manager is going to vote for his least preferred alternative. Each board member thus assumes the probability of the other members voting for their *least preferred* alternative to be 0. In addition, it is to be expected more likely that a group member will vote for a more preferred alternative. Plausible prior distributions are therefore monotone with regard to preferences. Every rational prior distribution in voting that is based on the single-vote criterion can be described by the function f_{SV}:[0;1] -> [0;1] where $f_{SV}(0)=0$, dependent on the normalized preferences V^*_m, as given in Formula (5). Furthermore, in plausible distributions, f_{SV} is monotone increasing.

(5) $$p^{a\ priori}_{m;SingleVote}(A_n) := \frac{f_{SV}(V^*_m(A_n))}{\sum_{l=1}^{N} f_{SV}(V^*_m(A_l))}$$

The decision is invariant to positive scaling of profit expectations and additive shifts in the distribution of personal interests. Profit expectations and personal interests are described by independent rectangular distributions over a given interval. The mean profit expectation therefore can be assumed to be 100, *mean* personal interests can be assumed to equal 0. Personal interests are parameterized as a multiple of the mean profit expectation of 100.

Figure 1: Overview of the basic model (without consideration of risk aversion)



It shows that varying the number of alternatives hardly makes any difference to the findings for N≥3, so that N=5 alternatives are assumed for simplification. If not stated differently, the evaluations are based on a reference situation with M=5 group members with naïve behaviour, an estimated standard deviation $s_{Estimate}$=10, a profit expectation range d_{Gew}=50, and personal interests amounting to one thousandth (PI_{max}=0,001) of the mean profit expectation of 100. Figure 1 presents an overview of the basic model.

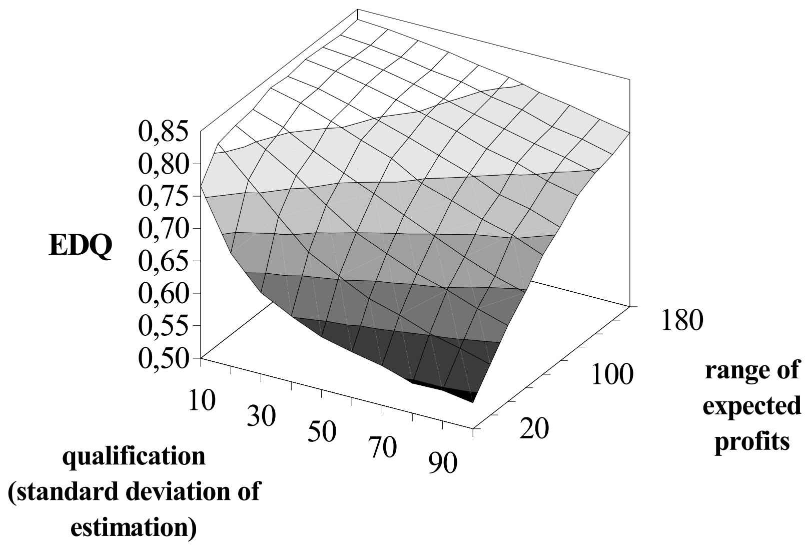
4 Results: influence of situation and control on decision quality

In this section, the influence on optimum bonus rate and expected decision quality (EDQ) as defined in section 2.1 by the interplay of eight different factors will be investigated. Excluding personal interests, EDQ would only be determined by differences in the profit expectations and predictive power of the board, thus group size and the qualification of the managers.

In this theoretical case in which a conflict of aims is missing, the bonus rate that is chosen should be as small as possible but big enough to guarantee a positive incentive for the lowest possible reward costs. Strictly speaking, there is not an optimum bonus rate. In the model the EDQ increases with decreasing bonus rate. However, at f=0 the EDQ reaches a point of discontinuity because the managers get the same utility from all alternatives. A very small bonus rate, however, results in almost no reward costs. Without a conflict of aims the managers orientate themselves only with regard to the reward available. Only ignorable low reward costs M·f are the result. Without a conflict of aims all managers have identical preferences after the interaction because their estimations of profit expectation are the same. Irrespective of strategic behaviour, all plausible voting procedures lead to the same conclusion.

Predictive power and EDQ are monotone increasing with increasing size of the board and qualification of the managers. In the model, the expected decision quality for fixed number of alternatives N and range of profit expectations $d_{Gew}(g)$ converges towards maximum EDQ when decisions are *always* made in the best interest of the shareholders (EDQ_{max} = 0,83 for N = 5 alternatives). The estimated standard deviation of the group after the interaction is $s_{Estimate}/M^{0,5}$. Irrespective of the particular parameters, EDQ thus depends only on the range of profit expectation and the ratio of estimated standard deviation to square root of group size (see Formula (3)). Figure 2 illustrates this connection.

Figure 2: Influence of qualification and range of expected profits on EDQ ($N=5$; $M=5$; $PI_{Max}=0$; always f_{opt}; naïve single vote ballot)



4.1 The interplay of personal interests, qualification, and reward

The incentive system

If the managers have personal interests in the alternatives, then the question arises as to what would be the optimum bonus rate f_{opt}. The shareholders should choose f so that, on the one hand, the conflict of aims between them and the managers is influenced as much as possible in their own common favour. On the other hand, they should only pay a limited amount of the difference in profit expected from the alternatives as reward. In an optimum situation, the utility the shareholders get by offering the managers an incentive is balanced by the cost of the reward they have to pay. In other words, the marginal utility in expected profit they get from paying the incentive corresponds to the marginal cost of paying the reward.

The greater the importance of personal interests, the more f_{opt} increases at first because that higher amount has to be compensated for. If personal interests become even more important, f_{opt} decreases again because more compensation would be too expensive. The importance of personal interests and the effect of reward costs on EDQ are also related to differences in profit expectation. The greater the range, the more the optimum bonus rate increases at first because the more profit would be available for paying rewards. However, the bonus rate will start to decrease again the more the alternatives begin to differ with regard to the partial utility of the reward available from them. In the end, only a small bonus rate is necessary for compensating personal interests.

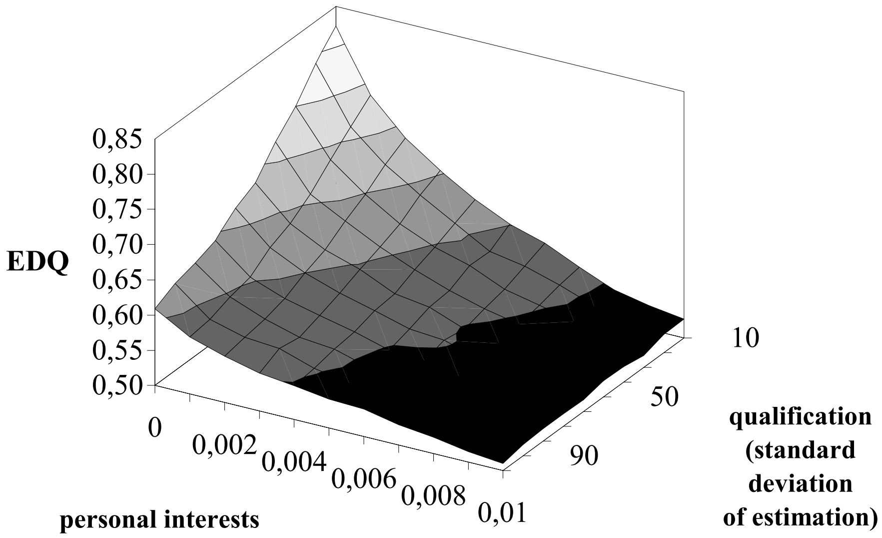
By and large, the combination of personal interests and range of profit expectation is decisive. If qualification declines, the incentive effect of a given bonus rate will decline because the managers' profit expectations are less precise, so that the incentives' effect has a stronger bias. The utility of an incentive and its marginal cost thus balance each other out at a smaller bonus rate. So, the optimum rate decreases with increasing qualification.

Expected decision quality

Personal interests also have a negative effect on EDQ when choosing the optimum bonus rate. Where there are strong personal interests, differences in profit expectation are decisive for the rate of decline in EDQ. The greater the range of profit expectation $d_{Gew}(g)$, the better increasing PI_{max} can be compensated for by paying bonuses, and EDQ declines at a slower rate.

If, due to personal interests, managers only decide partly in accordance with their estimations of profit expectation, then their predictive power has lower importance. If the conflict of aims increases with importance of personal interests, then the influence of qualification decreases. This connection is illustrated in Figure 3. At smaller PI_{max} the EDQ varies a lot more with qualification than at bigger PI_{max}.

Figure 3: Influence of personal interests and qualification on EDQ
(N=5; M=5; d_{Gew}=50; always f_{opt}; naïve single vote ballot)

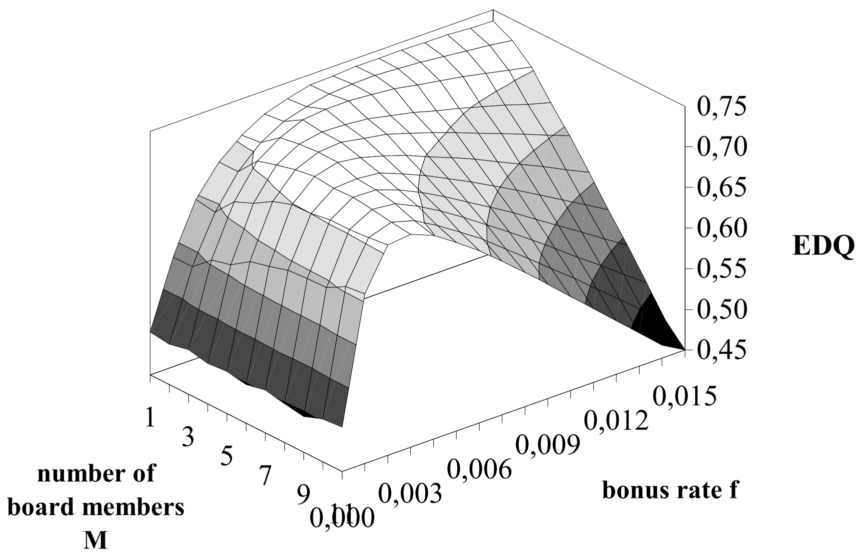


4.2 The importance of the number of board members

The incentive system

In bigger management groups it is more expensive for the shareholders to compensate for personal interests with bonuses. When f is increased, marginal reward cost increases in proportion to group size M. The marginal utility of the incentive does not, however, increase. Marginal utility and marginal cost would thus balance each other out at a lower level, so that the optimum bonus rate decreases with increasing M. Figure 4 shows this connection: if the group size M increases, the partial maximum is at smaller bonus rates f. The larger the group, the steeper the decline in EDQ for bonus rates beyond the optimum ($f>f_{opt}$) as the payment of too high a bonus rate has a stronger effect on EDQ in big groups, as described above.

Figure 4: Influence of number of board members and bonus rate on EDQ ($N=5$; $d_{Gew}=50$; $s_{Estimate}=10$; $PI_{Max}=0$; naïve single vote ballot)



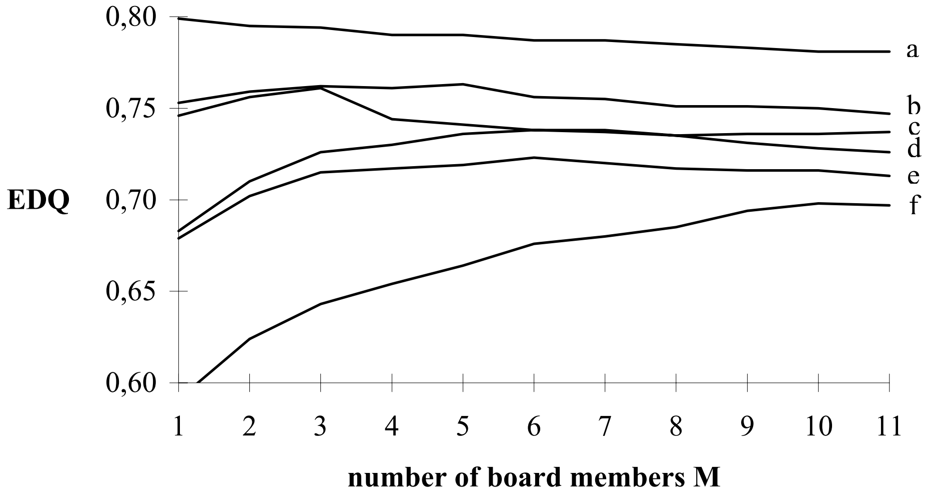
Expected decision quality

The answer provided by social psychology as to what is the optimum group size is often given irrespective of the specific situation [20], neglecting the following reasoning:

On the one hand, an increase in the size of a group comprising group members of low qualification increases the predictive power of the group more than would an increase in the size of a group comprising group members of high qualification.

On the other hand, the greater the importance of personal interests, the greater the tendency towards smaller groups first because payment of bonuses becomes more expensive, and second because the influence of qualification is smaller. When conflicts of aims and prediction occur simultaneously the optimum group size can only be determined according to the situation. Figure 5 illustrates the situational dependency of the model's optimum number of board members by presenting EDQ as a function of group size in six different parametric constellations (a to f). EDQ-optimal sizes of the board are 1 (a), 3 (d), 5 (b), 6 (c, e) and 11 (f).

Figure 5: Situational optimal number of board members (N=5; d_{Gew}=50; always f_{opt}; naïve single vote ballot; a to f: PI_{Max} = 0.001; 0.002; 0.002; 0.003; 0.003; 0.001; a to f: $s_{Estimate}$ = 10, 20, 40, 20, 40, 100)



4.3 The influence of strategic behaviour

If managers vote strategically, then they take the preferences of the other managers into consideration. Whether this leads to a concrete improvement or decline in decision quality from the shareholders' point of view depends on the managers' individual preferences. If a manager takes the other managers' preferences into consideration when he votes then, providing all other conditions remain the same, he will be more likely to vote for an alternative that is preferred by most of the others because in that case there is more chance of this alternative being chosen. Conversely, he is less likely to vote for an alternative that other managers judge to be bad because this has only a small chance of being chosen. On average, therefore, strategic behaviour results in those alternatives that best fulfil the *common* aims of the group members.

If there is no correlation between the personal interests of the group members, but all members have a positive utility for financial reward, then their assessments of the alternatives' attractiveness referring to the achievable reward will be the same after interaction. In this case, strategic voting promotes the importance of the reward aim compared to the importance of personal interests. Contrary to the belief that, in such a case, strategic behaviour has a negative influence on decision quality, the latter actually improves *on average* because the payment of bonuses is connected in a positive way to the shareholders' common goal of making as much profit as possible [11]. If, however, the managers have a common subgoal that is negatively correlated with the shareholders' goal, then the opposite effect takes place.

In the model, the positive effect of strategic voting on EDQ is only small. Table 1 shows the approximate EDQ for naïve and strategic voting behaviour when normalized preferences are taken as prior estimations for the other managers' voting behaviour. In spite of small differences and unclear statistical significance, the consistency of the results in all simulations indicates the validity of the qualitative statement.

Table 1: Influence of strategic behaviour on EDQ
(d_{Gew}=50; M=5; $s_{Estimate}$=10; PI_{Max}=0.001, f_{opt}, N=5; single vote ballot)

	PI_{max}			f			$s_{Estimate}$			d_{Gew}(g)			M (N=3)		
EDQ	.001	.004	.010	.002	.005	.008	10	40	100	50	100	200	3	5	7
Naïve	.732	.625	.529	.690	.732	.712	.732	.656	.574	.732	.787	.813	.681	.662	.660
Strat	.736	.632	.530	.696	.736	.714	.736	.658	.575	.736	.788	.815	.683	.668	.662

The effect of changes in prior distribution for the other managers' voting behaviour

So far, strategic behaviour has been investigated using the other managers' normalized preferences as prior distributions for the others' voting behaviour. The increase in EDQ is, however, not just valid for this type of prior distribution. The result actually seems to be fairly robust towards changing the assumed prior distribution, at least as long as the distributions are plausible in the sense of Formula (5). Table 2 shows EDQ and the percentage of naïve Bayesian Nash equilibria for ten different prior estimations of distribution following naïve, naïve Bayesian and strategic behaviour. The following ten functions for f_{SV} in Formula (5) are examined: 1/(N-1), $(\cdot)^{0.25}$, $(\cdot)^{0.5}$, ln(·+1), identity, $e^{\cdot}$-1, $(\cdot)^2$, $(\cdot)^3$, $(\cdot)^4$, and naïve prior estimation of distribution (most favoured preference with probability of 1). By and large, the influence of the prior on EDQ is so small that it can be ignored, at least for plausible prior distributions.

Table 2: Dependency of EDQ with strategic behaviour on priors
(N=5, d_{Gew}=50, M=5, $s_{Estimate}$ = 10; PI_{Max} = 0.001; always f_{opt}; single vote ballot,)

f_{SV}	$(N-1)^{-1}$	$(\cdot)^{0,25}$	$(\cdot)^{0,5}$	Ln(·-)	ident.	$e^{\cdot}$-1	$(\cdot)^2$	$(\cdot)^3$	$(\cdot)^4$	naïve
Naïve					.732					
Naïve Bayesian	.733	.733	.735	.735	.734	.735	.735	.735	.736	.736
Strategic	.735	.735	.736	.736	.736	.736	.736	.736	.737	.737
% nb.-equilibria	82%	83%	83%	83%	83%	85%	86%	88%	89%	89%

4.4 The managers' risk aversion

Adaptation in the model

It is now assumed that the managers' absolute risk aversion is constant. Otherwise, the wealth of the individual managers would have to be taken into consideration. The profit made from an alternative is assumed to be normally distributed with variance $s^2(A_n)$, the estimation of which is analogous to that of the expected profit. The errors of estimation are independently, identically, normally distributed with an estimated standard deviation $s_{Estimate}$, $s^2_m(A_n) \sim N(s^2(A_n);s^2_{Estimate})$. The parameterization is again done using the average and range of a rectangular distribution, from which the variances are drawn independently.

At a bonus rate f, the expected reward of choosing A_n for a manager is $f \cdot g(A_n)$, and its variance is $f^2 \cdot s^2(A_n)$. Manager m orients himself according to his subjective estimations $f \cdot g_m(A_n)$ and $f^2 \cdot s_m^2(A_n)$. Constant, absolute risk aversion results in an exponential utility function, the expected value of which, due to the normally distributed target size, exhibits only linear dependence on expectation and variance in target size distribution. In the case of personal interests, the result is (6) for preference $V_m(A_n)$ of manager m for A_n [17]. The right-hand side represents the certainty equivalent. In the following let $a^{risk} := 0.5 \cdot r_a$.

$$(6) \qquad V_m(A_n) = f \cdot g_m(A_n) - 0{,}5 \cdot r_a \cdot f^2 \cdot s^2_m(A_n) + PI_m(A_n) .$$

Effects on expected decision quality and optimal control

With risk aversion, the conflict of aims between shareholders and managers extends partly to the utility of the reward due to the necessity of risk sharing. The managers demand a risk premium, and the incentive effect of a given bonus rate will decrease. When risk aversion increases, the optimum bonus rate decreases. At the same time, the negative influence of personal interests on EDQ increases because the shareholders are less able to effectively compensate the personal interests of the risk averse managers financially.

When the conflict of aims is greater due to greater risk aversion, qualification becomes less significant, i.e. with increasing risk aversion, EDQ is less dependent on qualification. It then follows that a smaller group size would be better: as already described above, optimum group size is determined by a balance between the positive effects of increasing group size for improving predictive power and the negative effects of a greater conflict of aims with increasing group size due to the payment of bonus rates becoming more expensive. With increasing the risk aversion, the positive effect of an increase in group size on predictive power remains the same compared to risk neutrality. At the same time, the negative influence of personal interests increases due to the incentives becoming more expensive (risk premium), as discussed. Positive and negative effects on EDQ of an additional group member thus already balance themselves out in smaller groups.

4.5 The managers' tendency to conform

Social psychology emphasizes the influence of conformity on group decisions. From the point of view of conformity, alternatives that are favoured by many group members are attractive.

Adaptation in the model

In order to illustrate conformity, a component is incorporated into the utility function. Conformity is regarded as a subgoal that the managers take into consideration with their other goals. Again it is assumed that, *given the utility of the other managers*, the conformity of a manager in choosing an alternative can be quantified as a cash equivalent. Conformity is regarded as a component of personal interests and depends on the other group members' certainty equivalents for this alternative. a^{conf} where $0 \leq a^{conf} < 1$ represents the conformity component of personal interests in the board, $(1-a^{conf})$ corresponds to independent personal interests, represented in the following by $PI^0_m(A_n)$. Equation (7) is used to determine $PI_1(A_n)$ for the case of two group members m=1,2:

$$(7)\quad PI_1(A_n) = (1-a^{conf}) \cdot PI^0_1(A_n) + a^{conf} \cdot \left(f \cdot g(A_n) - a^{risk} \cdot [f^2 \cdot s^2(A_n)] + PI_2(A_n) \right)$$

When there are more than two group members, it is unclear with which ones a manager will tend to conform. Conformity could depend, for example, on sympathy or hierarchy. Here it is assumed that the tendency of each manager to conform is equally divided among the other board members. The certainty equivalent of each manager thus influences the personal interests of the other managers to the extent represented by $a^{conf}/(M-1)$. M>1 results in the equation (8) for all m=1, ...,M.

$$(8)\quad PI_m(A_n) = (1-a^{conf}) \cdot PI^0_m(A_n) + \frac{a^{conf}}{M-1} \sum_{\substack{l=1 \\ l \neq m}}^{M} f \cdot g(A_n) - a^{risk} \cdot f^2 \cdot s^2(A_n) + PI_l(A_n)$$

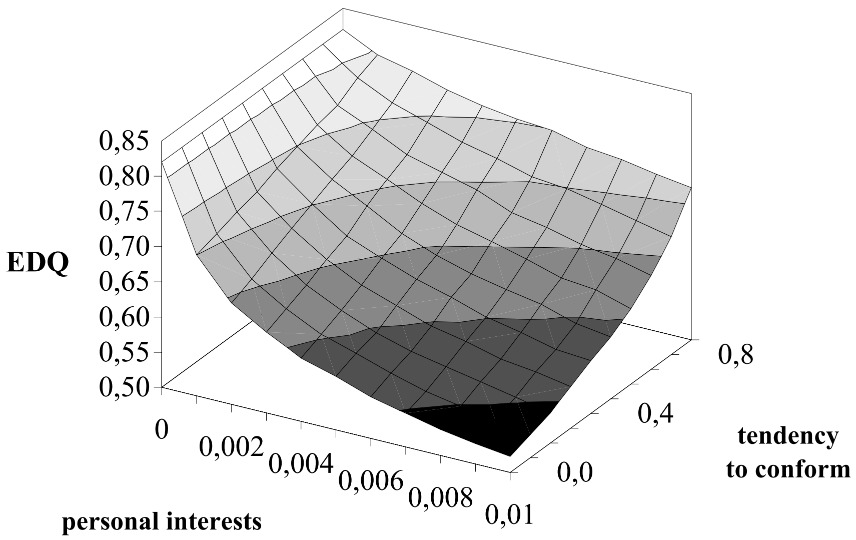
It is assumed that the managers are aware of each other's tendencies a^{conf}. In order to be able to establish personal interests consistently, rational expectations regarding the conformity of the group are necessary. Manager A must be able to anticipate the conformity of manager B with him, and vice versa. In order to calculate the personal interests of one manager, it is necessary to know the certainty equivalents of all the other managers. These certainty equivalents depend however on the personal interests of the first-mentioned manager. In consideration of the influence of conformity, consistent certainty equivalents for every A_n can be calculated from Formula (8) by solving the following linear set of equations for $PI_m(A_n)$ with M rows, which can always be solved for $a^{conf} < 1$. For all m=1,…,M:

$$(9)\quad PI_m(A_n) - \frac{a^{conf}}{M-1} \sum_{\substack{l=1 \\ l \neq m}}^{M} PI_l(A_n) = (1-a^{conf}) PI^o_m(A_n) + \frac{a^{conf}}{M-1} \sum_{\substack{l=1 \\ l \neq m}}^{M} f \cdot g(A_n) - a^{risk} f^2 s^2(A_n)$$

Effects of conformity on expected decision quality and optimal control

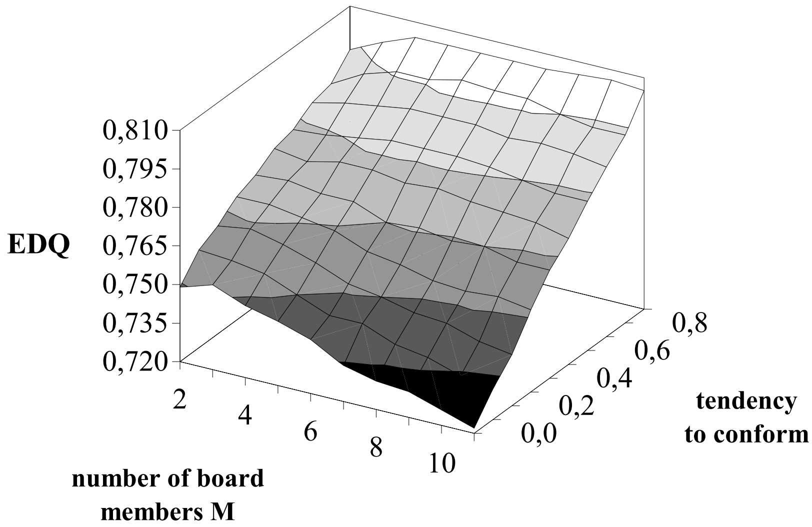
Where there is conformity in a group, the conflict of aims *within* the group is smaller. Should the common subgoal of the managers be in conflict with the goal of the shareholders, the conflict of aims *between managers and shareholders* will increase with increasing conformity, just as it will when the managers vote strategically. Should, however, the managers' only common subgoal be to get a bonus and should this subgoal positively correlate with the shareholders' goal, as in the model, then the conflict of aims between shareholders and managers will decrease and EDQ will increase with increasing conformity. In this situation, the negative influence of personal interests on EDQ decreases with an increasing tendency to conform, as illustrated in Figure 6.

Figure 6: Influence of tendency to conform on EDQ, depending on degree of personal interest (N=5; M=5; d_{Gew}=50; $s_{Estimate}$=10; always f_{opt}; naïve single vote ballot)



If it holds true that the conformity of a *given* group of managers is independent of its size, then *optimum* size increases with tendency to conform: as the importance of personal interests decreases, the improvement in predictive power with increasing group size will in terms of conflict only be negatively compensated for in a larger group. Accordingly, conformity has a stronger effect on EDQ in larger groups than in smaller ones (see Figure 7).

Figure 7: Influence of tendency to conform on EDQ, depending on number of board members ($N=5$; $d_{Gew}=50$; $s_{Estimate}=10$; $PI_{Max}=0{,}001$; always f_{opt}; naïve single vote ballot)



Empirical and social psychology investigations however indicate *diminishing* conformity with *increasing* size, which makes the above conclusions relative. The resultant effect of conformity on the optimal group size also depends on whether the relation between the shareholders' goal and the board's common subgoal is positive or negative, as described above.

4.6 The influence of the type of voting procedure

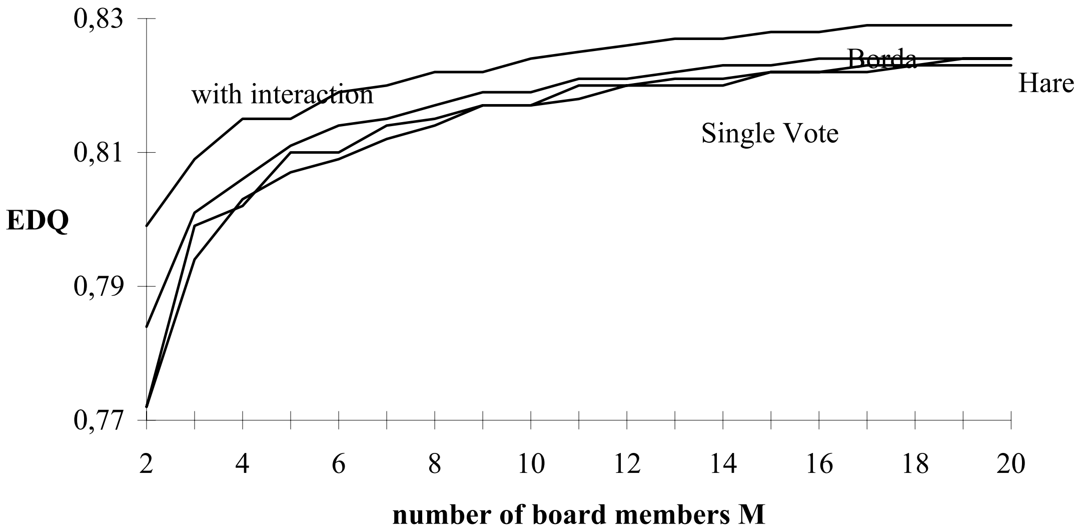
Using Borda's criterion and Hare's rule as a basis, the influence of a change in the type of voting procedure on EDQ will be investigated. When voting according to Borda's criterion, each group member awards N-1 points to one of the alternatives, N-2 points to another one, etc.; one alternative receives no points. The alternative with the highest number of points is chosen. When using Hare's rule, each board member submits a list of alternatives in order of preference. If an alternative achieves the absolute majority of first preferences ($> 0.5 \cdot M$), it is chosen. Otherwise, the alternative with the least number of first preferences is deleted from the preference ranking and the remaining alternatives move up a place. The procedure is then repeated for as long as it takes for one of the alternatives to achieve an absolute majority. In both cases, a suitable randomization is performed if required.

Decisive for the EDQ that results from using a voting procedure, providing all other conditions remain the same, is the amount of information about board members' preferences that the voting procedure allows for. The greater that amount of information, the higher the *mean* quality of the decision. Both Hare's rule and Borda's criterion result in a higher EDQ than the single-vote criterion because both comprise more information on the preference ranking of the managers.

Of these three types of voting procedure, Borda's criterion comprises the greatest amount of information because a complete ranking is put to the vote. Hare's rule comprises a medium information content. The effect of the information content of the voting procedures is greatest when there is *no* interaction on the managers' individual estimations of profit expectation (unlike assumed until know). Following an interaction, the managers' estimations of profit distribution are identical, so that a variation in the preference rankings can only come from personal interests. Without conflicting aims, all reasonable voting procedure lead to the same result after an interaction.

The effect of the information content of the voting procedures is shown in Figure 8 for a situation where there is no conflict of aims and no interaction. In accordance with the information content of the procedures, Borda's criterion results in the best EDQ, followed by Hare's rule and the single-vote criterion: individual estimations can be best taken into account when there is a group interaction.

Figure 8: Influence of voting procedure on EDQ, depending on number of board members ($N=5$; $d_{Gew}=50$; $s_{Estimate}=10$; $PI_{Max}=a^{risk}=a^{conf}=0$; $f_{opt}\equiv\varepsilon>0$; naïve single vote ballot)



Interestingly enough, Hare's rule is less suitable for small groups with even number of members than for uneven or bigger ones because if there is an equal number of votes for alternatives then random selection must be used. The absolute majority of first preferences required by Hare's rule is especially restrictive for small groups with even number of members and results in more frequent randomization and hence a deterioration of EDQ. The improved predictive power that is achieved by increasing the number of group members from three to four or five to six hardly translates into a better EDQ when using Hare's rule.

If there is a conflict of aims between the managers then their preferences will still differ following interaction due to varying personal interests. The level of information allowed for in the voting procedure is then decisive for how well personal interests can on average be compensated for by the bonus. In such a case the voting procedure can likewise be ranked according to their levels of information: the highest EDQ is achieved for Borda's criterion, followed by Hare's rule and the single-vote criterion.

It turns out that the more personal interests concur due to increasing conformity, the more alike the EDQs of the different voting procedures will become because the levels of information allowed for in the procedures have less of an effect on EDQ. When voting strategically using Borda's criterion or Hare's rule, suitable prior distributions for the other managers' voting of course have to be introduced. The results obtained when the managers vote strategically according to Borda's criterion and Hare's rule are similar to those obtained when voting strategically according to the single-vote criterion, whereby attention must be drawn to the problematic nature of statistical significance: It seems that strategic behaviour leads to a slight improvement in EDQ also for Borda's criterion and for Hare's rule, similar as in the single vote case. However, the improvement in EDQ is somewhat smaller for Borda's criterion and Hare's rule than for the single-vote rule. Therefore the type of voting procedure's influence on EDQ is smaller for strategic than for naïve voting behaviour. The types of voting procedure can however still be ranked according to their levels of information, with Borda's criterion beating the two others in terms of EDQ [10].

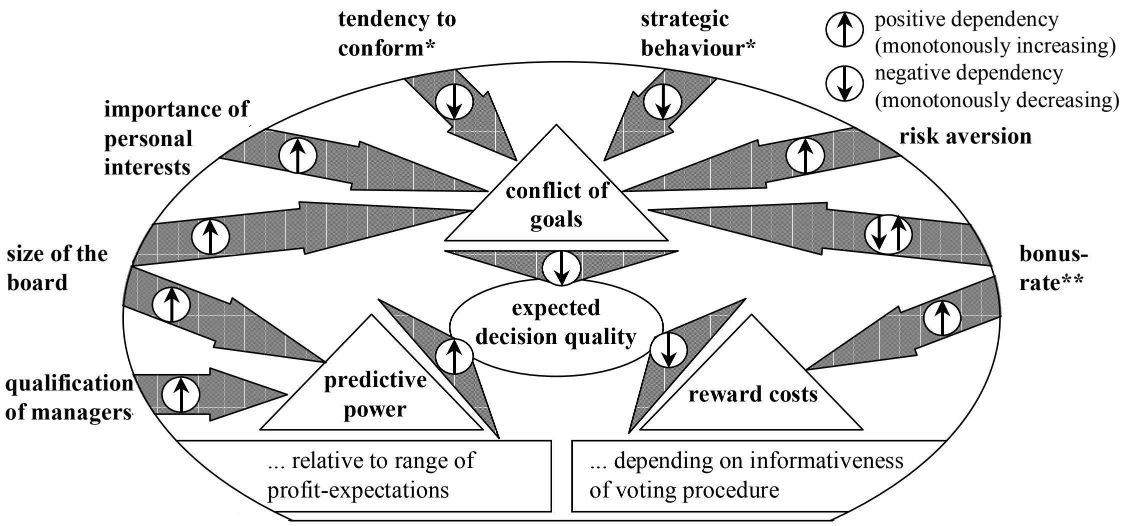
5 Conclusions

From a shareholder's point of view the quality of the decision made by a group of managers is dependent on numerous factors, which this paper investigated in detail with regard to their importance and type of influence. The primary influence of these factors on decision quality is determined mainly by three things, as described at the beginning of the paper: predictive power, conflicting goals and reward costs. Higher predictive power results in better decision quality, while a growing conflict of aims and increasing reward costs have a negative effect. Via one or two of these main determinants, the factors have varying effects on expected decision quality.

Higher qualification of the managers has a positive effect on decision quality due to an increase in predictive power. A greater importance of personal interests and, in the case of risk neutral shareholders, a greater risk aversion lead to a deterioration of decision quality due to a greater conflict of aims. A voting procedure such as Borda's criterion, that comprises a high degree of information about the managers' preferences, results in a high average decision quality. An increase in the size of the board has a positive effect on decision quality due to its greater predictive power, while at the same time increasing the conflict of aims within the group due to the payment of bonus rates being more expensive. Just as for an increase in bonus rate, the resultant effect in this case depends on the concrete problem. A higher bonus rate reduces the net quality of the decision due to the reward costs being more expensive. At the same time it has a positive effect on decision quality due to its primary effect of lessening the conflict of aims from conflicting interests. This is for risk averse managers however partly compensated for in a negative way by its secondary effect of increasing conflict of aims from sharing risk.

Strategic behaviour and tendency to conform reduce the conflict of aims within the group. This reduces the conflict of aims between shareholders and managers and thus increases decision quality, if a reward is paid as the managers' common subgoal and there are is no negative correlation between the shareholders' goal and a different common goal within the board. Figure 9 presents an overview of the primary mechanisms behind the interplay of influencing factors, main determinants and decision quality.

Figure 9: Primary EDQ-impact of influencing factors and mechanisms - overview



* reduces conflict of interests within the board only
** increasing bonus rate reduces personal interests conflict while increasing the conflict stemming from risk aversion

Besides the primary impacts on EDQ, there are of course secondary impacts, like the fact that a higher qualification of the managers also leads to a more effective impact of incentives paid in terms of the bonus rate. The secondary impacts that the influencing factors have on the effectiveness of each other are part of the discussion in section four.

In view of the growing influence of management groups investors would be wise not to just take the qualification and risk aversion of individual managers into account but also, when putting together a board of managers, to take into consideration their personal interests and the group-based factors like strategic behaviour and tendency to conform that would result from combining the individual characters.

Literature

[1] Arrow, K.J.: *Social Choice and Individual Values*. 2nd edition New York 1963.

[2] Aumann, R.: *Agreeing to Disagree*. In: Annals of Statistics 4 (1976), S.1236-1239.

[3] Gibbard, A.: *Manipulation of Voting Schemes*. In: Econometrica 41 (1973), S.587-601.

[4] Harsanyi, J.; Selten, R.: *A General Theory of Equilibrium Selection in Games*. Cambridge 1988.

[5] Holmström, B.: *Moral Hazard in Teams*. In: Bell Journal of Economics 13 (1982), S.324-340.

[6] Jaynes, E.T.: *Prior Probabilities*. In: IEEE Transactions on System Sciences and Cybernetics SSC-4 (1968), S.227-241.

[7] Laux, H.: *Grundfragen der Organisation*. Berlin 1979.

[8] Laux, H.: *Der Einsatz von Entscheidungsgremien*. Berlin 1979.

[9] Lehmann, E.L.: *Theory of Point Estimation*. New York 1983.

[10] Lindstädt, H.: *Optimierung der Qualität von Gruppenentscheidungen*. Heidelberg 1997.

[11] Miller, N.R.: *Graph-Theoretical Approaches to Voting*. In: American Journal of Political Science 21 (1977), S.769-803.

[12] Mookherjee, D.: *Optimal Incentive Schemes with Many Agents.* In: Review of Economic Studies 51 (1984), S.433-446.

[13] Nalebuff, B.; Stiglitz, J.: *Prices and Incentives: Towards a General Theory of Compensation and Competition.* In: Bell Journal of Economics 14 (1983), S.21-43.

[14] Nurmi, H.: *Comparing Voting Systems*. Dordrecht 1987.

[15] Pattanaik, P.K.: *Strategy and Group Choice*. Amsterdam 1978.

[16] Peleg, B.: *Game Theoretic Analysis of Voting in Committees*. Cambridge 1984.

[17] Pratt, J.: *Risk Aversion in the Small and in the Large*. In: Econometrica 32 (1964), S.122-136.

[18] Satterthwaite, M.A.: *Strategy-Proofness and Arrow's Condition*. In: Journal of Economic Theory 10 (1975), S.187-217.

[19] Schauenberg, B.: *Zur Logik kollektiver Entscheidungen*. Wiesbaden 1978.

[20] Türk, K.: *Gruppenentscheidungen - Sozialpsychologische Aspekte der Organisation kollektiver Entscheidungsprozesse*. In: Zeitschrift für Betriebswirtschaft 43 (1973), S.295-321.