HAGEN LINDSTÄDT

# VALUING OTHERS' INFORMATION UNDER IMPERFECT EXPECTATIONS 

A Cross-Individual Perspective on Harmful Information and Stock<br>Market Price Reactions


#### Abstract

Sometimes we believe that others receive harmful information. However, Marschak's value of information framework always assigns non-negative value under expected utility: it starts from the decision maker's beliefs - and one can never anticipate information's harmfulness for oneself. The impact of decision makers' capabilities to process information and of their expectations remains hidden behind the individual and subjective perspective Marschak's framework assumes. By introducing a second decision maker as a point of reference, this paper introduces a way for evaluating others' information from a cross-individual, imperfect expectations perspective for agents maximising expected utility. We define the cross-value of information that can become negative - then the information is "harmful" from a cross-individual perspective - and we define (mutual) cost of limited information processing capabilities and imperfect expectations as an opportunity cost from this same point of reference. The simple relationship between these two expected utility-based concepts and Marschak's framework is shown, and we discuss evaluating short-term reactions of stock market prices to new information as an important domain of valuing others' information.


KEY WORDS: value of information, decision under risk, imperfect expectations, cross-value of information, harmful information, stock market prices

JEL-CLASSIFICATION: D80, D82, D83.

## 1. INTRODUCTION

"If he had not been told about the opportunities in the Asian market, his investment strategy would have been much wiser."

Statements dealing with information we believe to be harmful can be heard from time-to-time. However, in the standard framework of assigning economic value to information, information can, at worst, be useless for an individual decision maker who maximises expected utility.

Since Marschak $(1954,1959)$ introduced his value of information framework, there appears to have been consensus about how to economically value information within the standard expected utility paradigm. Roughly, it values information with the gain a decision maker can extract from it in a specific decision due to a change in his subjective probabilities for given environmental states: a risk-neutral decision maker should value information vis-a-vis the money equivalent of the gain in utility he or she expects. Without bearing a cost for purchasing or processing information, this value can never be negative.

Valuation in the standard framework is thus clearly based on an individual and subjective perspective starting from the decision maker's subjective expectations: the impact of imperfect expectations and limited information processing capabilities is not made explicit but remains hidden behind the individual and subjective perspective that is assumed. Such imperfections of decision makers' individual expectations or bounds in the capability to process information as discussed by March (1978), Mongin and Walliser (1988), Lipman (1995), and Conlisk (1996) are usually not taken into account.

Two questions concerning the non-negativity and its connection to decision makers' expectations and information processing capabilities arise:
(1) Concerning non-negativity: In Marschak's framework, information is, at worst, irrelevant but never harmful. Of course, the reason for the non-negativity is precisely the fact that the decision maker values the information individually and subjectively. In most circumstances, we are usually not irritated or alarmed if someone calls an item of information harmful. How can this seeming contradiction be integrated into the framework without departing from expected utility? ${ }^{1}$
(2) Concerning imperfect expectations and information processing capabilities: Besides having incomplete information, economic agents face imperfect expectations and limitations in their information processing capabilities. Marschak's framework values information on the basis of any expectations a decision maker might have, provided they are consistent. What is the impact of imperfect expectations and limited information processing capabilities in valuing information, how can these limitations and imperfections be economically valued, and how do they influence non-negativity?

This paper offers an answer to these two questions by introducing a second decision maker as a point of reference. We define (asymmetrical) cross-values of information for two decision makers facing costly communication, imperfect expectations and limited information processing capabilities. A negative cross-value characterises inter-subjectively harmful information, and limited information processing capacities along with imperfect expectations are valued using opportunity cost. Information processing capabilities and expectations of the decision makers are reflected in their respective likelihood matrices for the specific information at hand.

In Section 2 we depart from Marschak's value of information. Section 3 defines the cross-value of information, while Section 4 deals with cost of limited information processing capabilities and imperfect expectations. The relationship between Marschak's individual value of information and the new approach makes up the discussion in Section 5. Section 6 sketches that the concept can be applied to evaluate stock market price reactions.

## 2. STARTING POINT: MARSCHAK'S VALUE OF INFORMATION

Marschak's (1954; 1959) framework values information about a specific decision under risk on the basis of expectations of the gain to be extracted due to a change in probabilities for given environmental states $S_{s}$. This is achieved by distinguishing between an item of information I and signals $I_{i}$, the deci-
sion maker believes to be possible. Throughout this paper, we restrict ourselves to risk-neutral decision makers maximizing expected payoff, a finite number of signals $I_{i}$, environmental states $S_{s}$, and acts $A_{a}$ with payoffs $U_{a s}$ to choose from.

The a priori value of the information $I=\left(I_{1} \ldots, I_{I}\right)$, before knowing the actual signal $I_{i}$, is the expected additional payoff the decision maker $\boldsymbol{\alpha}$ can get from the decision by possessing and processing the information, i.e., knowing the signal and its implications. To calculate this payoff, she has to judge the likelihoods in the information structure-matrix $\left(p_{\alpha}\left(I_{i} \mid S_{s}\right)\right)_{\text {is }}$ for the signals conditional on the possible states and work out the conditional probabilities $p_{\alpha}\left(S_{s} \mid I_{i}\right)$ using Bayes' Equation and $p_{\alpha}\left(I_{i}\right)$ using the total probability formula. The value of information equals the difference in expected payoff for the decision maker $\boldsymbol{\alpha}$ with and without the information.

If $a^{*}$ indexes the action with maximum expected payoff without the information, then the value of information is usually defined as in Equation (1): ${ }^{2}$

$$
\begin{align*}
& I V_{\alpha}\left(I,\left(p_{\alpha}\left(I_{i} \mid S_{s}\right)\right)_{i s}\right)=I V_{\alpha}(I) \\
& =\left[\sum_{i=1}^{I} p_{\alpha}\left(I_{i}\right) \times \max _{a=1, \ldots A}\left\{\sum_{s=1}^{S} p_{\alpha}\left(S_{s} \mid I_{i}\right) \times U_{a s}\right\}\right] \\
& \quad-\left[\sum_{s=1}^{S} p_{\alpha}\left(S_{s}\right) \times U_{a^{*} s}\right] \\
& = \\
& \quad \sum_{i=1}^{I} p_{\alpha}\left(I_{i}\right) \times\left[\max _{a=1, \ldots A}\left\{\sum_{s=1}^{S} p_{\alpha}\left(S_{s} \mid I_{i}\right) \times U_{a s}\right\}\right.  \tag{1}\\
& \left.\quad-\sum_{s=1}^{S} p_{\alpha}\left(S_{s} \mid I_{i}\right) \times U_{a^{*} s}\right] .
\end{align*}
$$

In the last row of Equation (1) the outer brackets for each $i$ contain the difference in expected payoff with and without the signal: If the signal turns out to be $I_{i}$, the brackets contain the a posteriori values of the signal $I_{i}$.

## 3. THE CROSS-VALUE OF INFORMATION

In most circumstances, we are not too much alarmed if someone tells us an item of information is harmful. At first, we may think of harmful information as something that is not objectively correct or true - and if a decision maker acts upon false information about the state of the world, thus choosing a disadvantageous action, we may think of him as a victim of false information.

Upon reconsideration, we may regard harmfulness of information as independent of its being true or false. Let us say you plan to buy a house, and based on the false information that it has rotten timbers, you decide not to buy it. In this case, the false information may well be advantageous to you, if you didn't know that the house also has a bad foundation. Vice versa, if you are prevented from buying a car by the true information that it is a lemon, this information could prove harmful in the unlikely case that there is a hidden treasure in its trunk worth more than what you paid to own the automobile.

In judging information to be useful or harmful, we have to refer to the effect it has on the decision's payoff. Again, there is a distinction between information that hindsight proves to be harmful - when the final environmental state is realised and information that one would call harmful from an a priori perspective. We are interested in the latter type.

By its own design, Marschak's value of information can never be negative. This follows directly from the last row in Equation (1): the decision maker maximises expected payoff conditionally, according to each signal $I_{i}$, by choosing the action $A_{a}$. Clearly, a rational decision maker will never choose an action that she expects to be sub-optimal. The a posteriori values of the signals $I_{i}$ are therefore all non-negative and, consequently, so is the a priorivalue of information for $I$.

Of course, the reason for non-negativity is precisely the fact that the decision maker herself individually and subjectively values the information based on her specific information processing capabilities and expectations. From her perspective,
every time she anticipates that an item of information could be harmful, she includes this in her valuation, i.e., in her individual likelihoods $p_{\alpha}\left(I_{i} \mid S_{s}\right)$. These likelihoods in consequence reflect limitations in the decision maker's information processing capabilities and imperfections in her expectations. To be consistent, a decision maker will by definition not believe that her own inferences regarding information are wrong while drawing these conclusions - otherwise, she would have come to different ones.

A decision maker's knowledge of the fact that her expectations are imperfect and her information processing capabilities are limited does not help her to improve her expectations or avoid the danger of drawing potentially wrong conclusions from any given item of information. If information is harmful to a decision maker, the harmfulness is caused precisely by her failure to anticipate its ability to harm.

In identifying information as harmful and by valuing it, we therefore need a useful point of reference. There are two fundamental ways to resolve this issue. The first would be to introduce objective likelihoods and probabilities in a deus ex machina fashion. This approach would lead to a unique point of reference from which information, expectations, and information processing capabilities could be valued "objectively." However, besides the principal difficulties one may have, assuming the existence of both objective and subjective probabilities in one framework, there would be no way to even principally judge the values we are interested in, as long as no omniscient individual is assumed to exist.

We will follow a second avenue for creating a point of reference. Instead of looking at a single decision maker $\alpha$, we posit a second individual $\beta$ who serves as a point of reference for $\alpha$ and vice versa. Just as one decision maker $\alpha$ can subjectively value the information she uses in her own decision, a second decision maker $\beta$ can subjectively value what he believes $\alpha$ gains or loses from having the information. In other words: we show how to value others' information. Akin to Marschak's idea of subjectively valuing information, we determine the cross-value of information for individual $\alpha$

## TABLE I

Decision under Risk with Subjective Probabilities

|  | $S_{1}$ |  |  | $S_{2}$ |
| :--- | ---: | ---: | :--- | ---: |
| $S_{3}$ |  |  |  |  |
| $p\left(S_{s}\right)$ | 0.5 | 0.3 | 0.2 | $E(\mathrm{U})$ |
|  |  |  |  |  |
| $A_{1}$ | 200 | 100 | -120 | 106 |
| $A_{2}$ | 150 | 150 | -10 | 118 |
| $A_{3}$ | 50 | 50 | 50 | 50 |

from $\beta$ 's perspective and vice versa. And we will call information harmful for $\alpha$ according to $\beta$, if $\beta$ believes the cross-value of the information for $\alpha$ is negative. In the remainder of this section, we introduce an example to the cross-value of information and derive a general definition.

EXAMPLE 1 The decision makers $\alpha$ and $\beta$ have to decide about which quantity of a certain good they should each produce. They distinguish among a high $\left(A_{1}\right)$, a medium $\left(A_{2}\right)$, and a low $\left(A_{3}\right)$ quantity.

The quantities yield different payoffs depending on the environmental states high $\left(S_{1}\right)$, medium $\left(S_{2}\right)$, or low $\left(S_{3}\right)$ demand that, by assumption, cannot be influenced by the decision makers and to which they assign probabilities (see Table I). Clearly, a risk-neutral decision maker would choose a medium production of quantity $A_{2}$, maximizing the expected payoff.

The decision makers are now independently offered a testmarket report containing imperfect information by showing low $\left(I_{3}\right)$, medium ( $I_{2}$ ) and high demand $\left(I_{1}\right)$ in the test market. $\alpha$ and $\beta$ share common probability beliefs for the $a$ priori state probabilities $p_{\alpha}\left(S_{s}\right)=p_{\beta}\left(S_{s}\right)=: p\left(S_{s}\right)$, but different beliefs due to differing information processing capabilities and expectations about the signal's likelihoods $p_{\alpha}\left(I_{i} \mid S_{s}\right)$ and $p_{\beta}\left(I_{i} \mid S_{s}\right)$, implying different $p_{\alpha}\left(I_{i}\right), p_{\beta}\left(I_{i}\right), p_{\alpha}\left(S_{s} \mid I_{i}\right)$ and $p_{\beta}\left(S_{s} \mid I_{i}\right)$. To ensure consistency, we assume that the decision
makers know each other's respective probability beliefs but cannot costlessly form identical expectations and beliefs about their respective probabilities due to cost of communication. ${ }^{3}$ Their probability beliefs are shown in Tables II. Table III contains the expected payoffs of the actions for each signal.

Clearly, both $\alpha$ and $\beta$ believe that a high, a medium or a low demand in the test market gives a certain indication for the respective demand in the overall market. However, $\alpha$ and $\beta$ differ in their beliefs of the extent of this stochastic indication. Altogether, $\alpha$ believes the test-market demand to have a stronger indication than does $\beta$. We can easily calculate $\alpha$ 's and $\beta$ 's individual value of the test-market information using Equation (1) (all numbers are rounded to a convenient number of digits):

## TABLE II

Subjective conditional probabilities of $\alpha$ and $\beta$

| $p_{\alpha}\left(I_{i} \mid S_{s}\right)$ | $I_{1}$ | $I_{2}$ | $I_{3}$ | $p\left(S_{s}\right)$ | $\overline{p_{\alpha}\left(S_{S} \mid I_{i}\right)}$ | $I_{1}$ | $I_{2}$ | $I_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | 0.80 | 0.15 | 0.05 | 0.5 | $S_{1}$ | 0.930 | 0.211 | 0.116 |
| $S_{2}$ | 0.10 | 0.80 | 0.10 | 0.3 | $S_{2}$ | 0.070 | 0.676 | 0.140 |
| $S_{3}$ | 0.00 | 0.20 | 0.80 | 0.2 | $S_{3}$ | 0.000 | 0.113 | 0.744 |
| $p_{\alpha}\left(I_{i}\right)$ | 0.430 | 0.355 | 0.215 |  |  |  |  |  |
| $p_{\beta}\left(I_{i} \mid S_{s}\right)$ | $\mathrm{I}_{1}$ | $I_{2}$ | $I_{3}$ | $p\left(S_{s}\right)$ | $\overline{p_{\beta}\left(S_{s} \mid I_{i}\right)}$ | $I_{1}$ | $I_{2}$ | $I_{3}$ |
| $S_{1}$ | 0.50 | 0.25 | 0.25 | 0.5 | $S_{1}$ | 0.694 | 0.352 | 0.439 |
| $S_{2}$ | 0.30 | 0.50 | 0.20 | 0.3 | $S_{2}$ | 0.250 | 0.423 | 0.211 |
| $S_{3}$ | 0.10 | 0.40 | 0.50 | 0.2 | $S_{3}$ | 0.056 | 0.225 | 0.351 |
| $p_{\beta}\left(I_{i}\right)$ | 0.360 | 0.355 | 0.285 |  |  |  |  |  |

TABLE III
Actions' conditional expected payoffs of $\alpha$ and $\beta$
Decision maker $\alpha$
Decision maker $\beta$

| $E_{\alpha}\left(U\left(A_{a}\right)\right)$ | $I_{1}$ | $I_{2}$ | $I_{3}$ | $E_{\alpha}\left(U\left(A_{a}\right)\right)$ | $I_{1}$ | $I_{2}$ | $I_{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $p_{\alpha}\left(I_{i}\right)$ | 0.430 | 0.355 | 0.215 | $p_{\beta}\left(I_{i}\right)$ | 0.360 | 0.355 | 0.28 |
| $A_{1}$ | 193.0 | 96.3 | -52.1 | $A_{1}$ | 157.1 | 85.6 | 66.7 |
| $A_{2}$ | 150.0 | 132.0 | 30.9 | $A_{2}$ | 141.1 | 113.9 | 93.9 |
| $A_{3}$ | 50.0 | 50.0 | 50.0 | $A_{3}$ | 50.0 | 50.0 | 50.0 |

$$
\begin{align*}
I V_{\alpha}(I)= & 0.430 \times(193-150)+0.355 \times(132-132) \\
& +0.215 \times(50-30.9)=22.60 \tag{2}
\end{align*}
$$

$$
\begin{equation*}
I V_{\beta}(I)=(157.1-141.1) \times 0.36=5.76 \tag{3}
\end{equation*}
$$

We are interested in the a priori value that decision maker $\beta$ believes the market test has for $\alpha$. For a high test-market demand $\left(I_{1}\right), \alpha$ changes her choice from $A_{2}$ to $A_{1}$, gaining $157.1-141.1=16.0$ in expected ( a posteriori) payoff from $\beta$ 's perspective. For the signal $I_{2}$ (medium test-market demand), $\alpha$ does not change her choice relative to not having the information, but sticks to $A_{2}$, resulting in no a posteriori gain or loss in payoff. For a low test-market demand $I_{3}, \alpha$ (in $\beta$ 's judgment sub-optimally) changes from choosing $A_{2}$ without the information towards $A_{3}$, resulting in a change of $50.0-93.9=$ -43.9 in expected payoff (according to $\beta$ ). From the a priori perspective, i.e., before knowing the signal, $\beta$ calculates the value of the information for $\alpha$ using these a posteriori values and his own probabilities for the signals. Altogether, $\beta$ judges the mutual value of the information for $\alpha$ as being

$$
\begin{align*}
I V_{\alpha ; \beta}(I)= & 0.360 \times(157.1-141.1)+0.355 \times 0.0 \\
& +0.285 \times(50.0-93.9)=-6.75 \tag{4}
\end{align*}
$$

So from $\beta$ 's perspective, $\alpha$ would be better off without the information. Clearly, $\beta$ believes that the information is harmful for $\alpha$ : the small gain in payoff $\beta$ expects for $\alpha$ under $\mathrm{I}_{1}$ is negatively more than compensated for by the loss attributable to $\alpha$ 's "false" conclusion of switching from a medium to a low production quantity (from $A_{2}$ to $A_{3}$ ) when $I_{3}$ is realized.
$\beta$ himself would only change his preferred action from $A_{2}$ if there were a high test-market demand ( $I_{1}$ ). From $\alpha$ 's perspective, this results in an a posteriori change in expected payoff of $193.0-150.0=43.0$. For $I_{2}$ and $I_{3}, \beta$ would stick with $\mathrm{A}_{2}$, consequently not changing his payoff expectation in $\alpha$ 's or
in his own judgment. From an a priori perspective, $\alpha$ believes the information for $\beta$ to have a mutual value of

$$
\begin{align*}
I V_{\beta ; \alpha}(I)= & 0.430 \times(193.0-150.0)+0.355 \times 0.0 \\
& +0.215 \times 0.0=18.49 \tag{5}
\end{align*}
$$

In conclusion, $\alpha$ believes that it is better for $\beta$ to have the information than not to have it, although the information is of slightly less value to $\beta$ than it is to $\alpha$ herself, since $\alpha$ 's subjective valuation of the information is 22.6.

We assume finite numbers of actions $A_{a}$, states $S_{s}$, signals $I_{i}$, and two risk-neutral decision makers $\alpha$ and $\beta$ with the same payoffs $U_{a s}$, common state priors $p\left(S_{s}\right)$ who know each others' probability beliefs for the signals' conditional likelihoods and for whom communication is costly.

DEFINITION 1 Let $a^{*}$ index the action that both decision makers prefer without the information and $a_{i ; \alpha}^{*}$ index the action decision maker $\alpha$ prefers when knowing signal $I_{i}$. Using these assumptions, the cross-value of information I for decision maker $\alpha$ as valued by $\beta$ is defined as:

$$
\begin{align*}
I V_{\alpha ; \beta}(I)= & \sum_{i=1}^{I} p_{\beta}\left(I_{i}\right) \times\left[\left(\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a^{*} ;, \alpha^{s}}\right)\right. \\
& \left.-\left(\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a^{* s}}\right)\right] \tag{6}
\end{align*}
$$

RESULT 1 The cross-value of information can become negative (see above).

## 4. THE CROSS-COST OF LIMITED INFORMATION PROCESSING CAPABILITIES AND IMPERFECT EXPECTATIONS

Decision makers have incomplete bases of information, limited capabilities to process information and imperfect expectations
about states of the world. We will again consider a certain decision under conditions of risk and changes in subjective priors for environmental states stemming from an item of information. Again, a decision maker's consciousness of her own limitations in information processing and imperfections in building expectations will not help her to form a better judgment. As in the previous section, we need a point of reference and will therefore stick with a second decision maker.

It is important to note that cross-valuing limitations in information processing capabilities and imperfections in expectations is different from that of calculating a cross-value of information for two reasons. First, the two approaches use different comparisons. In order to cross-value information, one decision maker values the difference in expected payoff the other gets with and without information. For cross-valuing the limitations and imperfections, one decision maker evaluates the difference in expected payoffs that stems from the two decision makers' different processing capabilities and expectations for conditional likelihoods.

Second, it does not make much sense to ask for the "value of limited information processing capabilities and imperfect expectations" as a gain. Instead, imperfections and limitations are typically accounted for by calculating their cost, namely the opportunity cost they cause. Roughly, these opportunity costs are determined by the difference in expected payoffs that stems from the different expectations using an a priori perspective. ${ }^{4}$

EXAMPLE 2 We stick with the test-market decision from Example 1 as introduced in Tables I-III. We assign mutual opportunity cost to the decision makers' limited information processing capabilities and imperfect expectations that manifest themselves in their respective subjective conditional likelihoods shown in Table II. As in Section 3, we start from the a posteriori values for the respective signals $I_{i}$. Let us first assume that there will be low demand in the test market, i.e., the signal turns out to be $I_{3}$. In that case, the two individuals choose differently (see Table III): $\beta$ would choose
to produce a medium quantity $\left(A_{2}\right)$ with a revised, a posteriori expected payoff of 93.9 , while $\alpha$ would switch to producing only a low quantity $\left(A_{3}\right)$, expecting a payoff of 50 . If the test-market demand is high or medium ( $I_{1}$ or $I_{2}$ ), the two individuals still expect different a posteriori payoffs. However, their different processing capabilities and expectations would not lead to different actions: both would choose $A_{1}$ when $I_{1}$ and $A_{2}$ when $I_{2}$ is the signal from the test market.

From $\beta$ 's perspective, the situation looks like this: if $I_{1}$ or $I_{2}$ turns out to be the signal, $\alpha$ holds different beliefs about the revised expected payoff, due to her specific processing limitations and different expectations for the test-market information. However, these different beliefs do not lead to a choice differing from $\beta$ 's own beliefs. Therefore, $\alpha$ 's specific processing limitations and expectations in her conditional choice under $I_{1}$ and $I_{2}$, do not imply any actual difference relative to $\beta$ 's. In case of $I_{3}, \alpha$ wouldn't choose $\beta$ 's preferred action $A_{2}$, but rather $A_{3}$ according to her own beliefs. From $\beta$ 's perspective this choice, conditional under $I_{3}$, implies an a posteriori loss in expected payoff of $93.9-50.0=43.9$, the a posteriori cost of $\alpha$ 's limited information processing capabilities and her imperfect expectations in $\beta$ 's judgment.

The a priori cross-cost of $\alpha$ 's limited processing capabilities and imperfect expectations from $\beta$ 's perspective, i.e., before knowing the signal, is the expected value of the respective $a$ posteriori values for all possible signals. Decision maker $\beta$, of course, uses his own beliefs for probabilities and payoffs in the calculation. So the a priori cross-cost of $\alpha$ 's limited processing capabilities and imperfect expectations for the market test from $\beta$ 's perspective in the example is ${ }^{5}$

$$
\begin{align*}
C E_{\alpha ; \beta}(I) & =0.360 \times 0.0+0.355 \times 0.0+0.285 \times(93.9-50.0) \\
& =12.51 \tag{7}
\end{align*}
$$

From $\alpha$ 's perspective, the situation looks very similar. $\beta$ 's deviating beliefs do not imply any choice that would reduce his a posteriori payoff expectation for $I_{1}$ and $I_{2}$. For $I_{3}, \beta$ would choose $A_{2}$, resulting in a loss of $50.0-30.9=19.1$ in $a$
posteriori expected payoff. Decision maker $\alpha$ naturally calculates the cost of $\beta$ 's limited processing capabilities and imperfect expectations for the test-market information, using $\alpha$ 's beliefs about both the expected payoffs and the probabilities involved:

$$
\begin{align*}
C E_{\beta ; \alpha}(I) & =0.430 \times 0.0+0.355 \times 0.0+0.215 \times(50.0-30.9) \\
& =4.11 \tag{8}
\end{align*}
$$

We now define the cross-cost of limited processing capabilities and imperfect expectations formally, using the same assumptions as in Definition 1:

DEFINITION 2 Let $a_{i ; \alpha}^{*}$ index the action $\alpha$ would choose to maximize her expected payoff, once she knows that signal $I_{i}$ has been realized. The cross-cost of $\alpha$ 's limited information processing capabilities and imperfect expectations for information I as valued by $\beta$ is then defined as:

$$
\begin{align*}
C E_{\alpha ; \beta}(I)= & \sum_{i=1}^{I} p_{\beta}\left(I_{i}\right) \times\left[\max _{a=1, \ldots, A}\left\{\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a s}\right\}\right. \\
& \left.-\left(\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a^{*} ;, \alpha^{s}}\right)\right] \tag{9}
\end{align*}
$$

For each $i$, the term within the outer brackets is the a posteriori cross-cost of the processing limitations and imperfect expectations for that signal $i$ : for this $i$, the payoff that is expected when $\alpha$ 's choices are weighted with $\beta$ 's conditional probabilities is subtracted from $\beta$ 's expected payoff using $\beta$ 's own choices.

Clearly, this cost can never be negative: if, for a certain signal, $\alpha$ chooses the same action as $\beta$, the term within the outer brackets is zero for that $i$, but can never become negative. Therefore, the same is true for $C E_{\alpha ; \beta}(I)$. This non-negativity corresponds to the fact that, starting from $\beta$ 's perspective, $\alpha$ can, at best, decide as well as $\beta$, but never better.

RESULT 2 Under the assumptions stated above, the mutual cost of a decision maker's imperfect expectations is non-negative: $C E_{\alpha ; \beta}(I) \geq 0$.

## 5. THE RELATIONSHIP BETWEEN THE THREE CONCEPTS

There is a very simple connection linking the three values measuring the individual to the cross-value of information and to the cross-cost of limited processing capabilities and imperfect expectations: the cross-value of information $I$ for decision maker $\alpha$ from decision maker $\beta$ 's perspective equals the difference between the individual value of that information for decision maker $\beta$ and the cross-cost of $\alpha$ 's limited processing capabilities and her imperfect expectations for $I$ in $\beta$ 's judgment:

RESULT 3 With the previous assumptions and notations, for every decision, item of information I, and all risk-neutral decision makers $\alpha$ and $\beta$, we know:

$$
\begin{equation*}
I V_{\alpha ; \beta}(I)=I V_{\beta}(I)-C E_{\alpha ; \beta}(I) \tag{10}
\end{equation*}
$$

Proof. For all $i=1, \ldots I$, we show that the equation is true when using the respective a posteriori values within the outer brackets from (1) (last row), (6) and (9). From this, Equation (10) is directly implied as shown in the next equation. The last equality is immediately clear:

$$
\begin{aligned}
& (10) \quad \Leftrightarrow I V_{\alpha ; \beta}(I)-I V_{\beta}(I)+C E_{\alpha ; \beta}(I)=0 \Leftarrow \\
& =\left[\left(\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a_{i ; \alpha}^{*} s}\right)-\left(\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a^{*} s}\right)\right] \\
& \quad-\left[\max _{a=1, \ldots, A}\left\{\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a s}\right\}\right.
\end{aligned}
$$

$$
\begin{aligned}
& \left.-\left(\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a^{*} s}\right)\right] \\
& +\left[\max _{a=1, \ldots, A}\left\{\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a s}\right\}\right. \\
& \left.-\left(\sum_{s=1}^{S} p_{\beta}\left(S_{s} \mid I_{i}\right) \times U_{a_{i: \alpha}^{*}}\right)\right] \\
= & 0, \text { q.e.d }
\end{aligned}
$$

Comparing this general result with the numbers from the example we indeed find that, from $\beta$ 's perspective (see Equations (3), (4), and (7)):

$$
\begin{equation*}
I V_{\alpha ; \beta}(I)=I V_{\beta}(I)-C E_{\alpha ; \beta}(I):-6.75=5.76-12.51 \tag{11}
\end{equation*}
$$

and from $\alpha$ 's perspective (see Equations (2), (5), and (8)):

$$
\begin{equation*}
I V_{\beta ; \alpha}(I)=I V_{\alpha}(I)-C E_{\beta ; \alpha}(I): 18.49=22.60-4.11 \tag{12}
\end{equation*}
$$

Since the individual value of information is known to be always non-negative, and the same is true for the cross-cost of limited processing capabilities and imperfect expectations (see Result 2), it is clear that the cross-value any risk-neutral decision maker $\beta$ assigns to any item of information $I$, for a second such decision maker $\alpha$ can never exceed $\beta$ 's own individual valuation of this same information. From $\beta$ 's perspective $\alpha$ can, at best, draw the same conclusions from the information as $\beta$ does himself:

RESULT 4 Applying the previous assumptions and notation, for every decision, item of information I, and all risk-neutral decision makers $\alpha$ and $\beta$, we know:

$$
\begin{align*}
& I V_{\alpha ; \beta}(I) \leqslant I V_{\beta}(I)  \tag{13}\\
& I V_{\alpha ; \alpha}(I)=I V_{\alpha}(I)  \tag{14}\\
& C E_{\alpha ; \alpha}(I)=0 \tag{15}
\end{align*}
$$

The easy proof is omitted. Looking at the examples we have from $\beta$ 's perspective $5.76 \geq-6.75$ and from $\alpha$ 's perspective $22.60 \geq 18.49$ as special cases of (13).

## 6. REACTIONS OF STOCK MARKET PRICES AS AN APPLICATION

An important domain for applying the concept of valuing others' information under imperfect expectations is the capital market, especially short-term reactions of stock market prices: every time new and relevant information becomes known, stock market prices will react somehow. To anticipate this reaction, an investor not only has to judge the impact he expects for the company itself. In addition, each investor has to build a judgment about which impact the information will have on the "the market", i.e., on the other investors' expectations.

Imagine an investor $\beta$ receives important private information about an enterprise - say a merger between two companies - a day before this information becomes publicly known. For finding his best investment strategy it is eminent what conclusions other investors will draw once they receive the information. Should our investor $\beta$ believe that the merger will not payoff in the long run, he would still be better off buying the stock that day in case he anticipates the other investors will believe in positive consequences for the enterprise (besides the fact that this might be a case of insider trading).

In evaluating implications of information entering the stock market, it seems plausible that investors do not necessarily revise their probability judgments identically short term, while they may have developed similar priors long term. This contrast between valuing information from one's own perspective versus that of others is precisely what the cross-value of information concept deals with.

## 7. SUMMARY AND FINAL REMARKS

Usually one tries to avoid introducing an objective point of reference in information economics. Still, in many economically interesting situations, we struggle with a completely individual and subjective point of view because we cannot adequately describe many real-life phenomena. Therefore, we often introduce a second or third decision maker to illuminate a situation from more than one perspective, i.e., intersubjectively. The best-known examples deal with informational asymmetries between two (or more) economic agents. We have taken a similar approach - by introducing a second individual as a point of reference for the valuation of information. This enables us to value other agents' information by defining the concept of cross-value of information and cross-cost of limited information processing capabilities and imperfect expectations. The result also is a formal, inter-subjective definition of the common term "harmful information."

One important area of application is anticipating shortterm reactions of stock market prices to newly available information. Also, multi-layered decisions or those that involve three or more agents can be handled the same way by separately approaching each pair of decision makers that is of interest.

The framework is intended to contribute to the economic discussion by employing a convenient decision-theory framework to bridge our regard for models dealing with informational asymmetries to those considering the value of information.

## ACKNOWLEDGEMENTS

The author thanks Barbara Hobbie, Peter Kesting, Hugo Kossbiel, Annette Müller, Pierfrancesco La Mura, Thomas Spengler, Michael Wolff, and an anonymous referee for valuable comments.

## NOTES

1. Since Wakker's seminal paper (1988) it is well known that the value of information can become negative for decision makers who do not maximise expected utility. Grant et al. (2000) discuss the close connection between non-negativity and dynamic consistency.
2. Fundamental analyses of information, information processing and information structures appear in Hirshleifer and Riley (1979), McGuire (1986), and Lindstädt (2001). For a simpler notation, we can leave out the information structure indicated in (1), if it is clear from the context which information structure is referred to. Note that Equation (1) is guaranteed only for risk-neutral decision makers.
3. With costless communication, the decision makers could build identical probability beliefs for the signals' conditional likelihoods from their common priors, see Geanakoplos and Polemarchakis (1982). Notice that we are only assuming $\alpha$ and $\beta$ to know each other's probability beliefs, which is different from Aumann's assumption of these beliefs being common knowledge. As he himself pointed out, the assumption of knowing each other's probability beliefs is not sufficient for his famous result: "if two people have the same priors, and their posteriors for an event are common knowledge, then these posteriors are equal", see Aumann (1976). If the individuals differ in their information structures and therefore in their expectations, and if communication is costly, they might well have the same priors and different posteriors even though knowing each others' priors and posteriors.
4. We have to be aware of the fact that no decision maker will be able to calculate this opportunity cost for herself in a reasonable way that could be used to allocate information processing capacity for improving one's expectations. The problem lies in the allocation of a scarce resource, "information processing," when exactly that same resource is required to solve the allocation task. Taking this approach would make us run into the problem of infinite regress - see Mongin and Walliser (1988), Lipman (1991, 1995), and Conlisk (1996).
5. We calculate the required a priori values by starting from the a posteriori values and taking their mean using $p_{\beta}\left(I_{i}\right)$. Note that this follows a similar idea found in Marschak's individual value of information when starting from the posteriors. This calculation is valid in the case of risk-neutral decision makers, but usually not for other risk preferences.

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Address for correspondence: Hagen Lindstädt, Institute of Applied Economics and Management, Karlsruhe University (TH), Postfach 6980, D-76128 Karlsruhe, Germany. Tel.: +49-(0)721-608-3431;
E-mail: lindstaedt@ibu.uni-karlsruhe.de

